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On kaonic deuterium. Quantum field-theoretic and relativistic covariant approach

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Abstract. We study kaonic deuterium, the bound K^-d state A_{Kd}. Within a quantum field-theoretic and relativistic covariant approach we derive the energy level displacement of the ground state of kaonic deuterium in terms of the amplitude of K^-d scattering for arbitrary relative momenta. Near threshold our formula reduces to the well-known DGBT formula. The S-wave amplitude of K^-d scattering near threshold is defined by the resonances $\Lambda(1405)$, $\Sigma(1750)$ and a smooth elastic background, and the inelastic channels $K^-d \to NY$ and $K^-d \to NY\pi$, where $Y = \Sigma^{\pm}, \Sigma^0$ and Λ^0, where the final-state interactions play an important role. The Ericson-Weise formula for the S-wave scattering length of K^-d scattering is derived. The total width of the energy level of the ground state of kaonic deuterium is estimated using the theoretical predictions of the partial widths of the two-body decays $A_{Kd} \to NY$ and experimental data on the rates of the NY pair production in the reactions $K^-d \to NY$. We obtain $\Gamma_{1s} = (630 \pm 100)\,\mathrm{eV}$. For the shift of the energy level of the ground state of kaonic deuterium we predict $\epsilon_{1s} = (325 \pm 60)\,\mathrm{eV}$.

PACS. 11.10.Ef Lagrangian and Hamiltonian approach – 11.55.Ds Exact S matrices – 13.75.Gx Pion-baryon interactions – 36.10.-k Exotic atoms and molecules (containing mesons, muons, and other unusual particles)

1 Introduction

Kaonic deuterium A_{Kd} is an analogy of hydrogen with an electron and the proton replaced by the K^--meson and the deuteron, respectively. The relative stability of kaonic deuterium is fully due to Coulomb forces [1–7]. The Bohr radius of kaonic deuterium, is

$$a_{\mathrm{B}} = \frac{1}{\mu\alpha} = \frac{1}{\alpha}\left(\frac{1}{m_K} + \frac{1}{m_d}\right) = 69\,\mathrm{fm}, \qquad (1.1)$$

where $\mu = m_K m_d/(m_K + m_d) = 391\,\mathrm{MeV}$ is a reduced mass of the K^-d system, calculated at $m_K = 494\,\mathrm{MeV}$ and $m_p = 1876\,\mathrm{MeV}$ [8], and $\alpha = e^2/\hbar c = 1/137.036$ is the fine-structure constant [8]. Below we use the units $\hbar = c = 1$, then $\alpha = e^2 = 1/137.036$. Since the Bohr radius of kaonic hydrogen is much greater than the range of strong low-energy interactions $R_{\mathrm{str}} \sim 1/m_{\pi^-} = 1.42\,\mathrm{fm}$ and the radius of the deuteron $r_d = 4.32\,\mathrm{fm}$ [9], the strong low-energy interactions can be taken into account perturbatively [1–7].

According to Deser, Goldberger, Baumann and Thirring [1] the energy level displacement of the ground state of kaonic deuterium can be defined in terms of the S-wave amplitude $f_0^{K^-d}(Q)$ of low-energy K^-d scattering as follows:

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = \frac{2\pi}{\mu} f_0^{K^-d}(0)|\Psi_{1s}(0)|^2, \qquad (1.2)$$

where $\Psi_{1s}(0) = 1/\sqrt{\pi a_{\mathrm{B}}^3}$ is the wave function of the ground state of kaonic hydrogen at the origin and $f_0^{K^-d}(0)$ is the amplitude of K^-d scattering in the S-wave state,

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calculated at zero relative momentum $Q = 0$ of the K^-d pair. The DGBT formula can be rewritten in the equivalent form

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = 2\alpha^3\mu^2 f_0^{K^-d}(0), \quad (1.3)$$

where $2\alpha^3\mu^2 = 602\,\mathrm{eV\,fm^{-1}}$ and $f_0^{K^-d}(0)$ is measured in fm. The formula (1.3) is used by experimentalists for the analysis of experimental data on the energy level displacement of the ground state of kaonic deuterium [10].

For non-zero relative momentum Q the amplitude $f_0^{K^-d}(Q)$ is defined by

$$f_0^{K^-d}(Q) = \frac{1}{2iQ}\Big(\eta_0^{K^-d}(Q)e^{2i\delta_0^{K^-d}(Q)} - 1\Big), \quad (1.4)$$

where $\eta_0^{K^-d}(Q)$ and $\delta_0^{K^-d}(Q)$ are the inelasticity and the phase shift of the reaction $K^- + d \to K^- + d$, respectively. At relative momentum zero, $Q = 0$, the inelasticity and the phase shift are equal to $\eta_0^{K^-d}(0) = 1$ and $\delta_0^{K^-d}(0) = 0$. For $Q \to 0$ the phase shift behaves as $\delta_0^{K^-d}(Q) = a_0^{K^-d}Q + O(Q^2)$, where $a_0^{K^-d}$ is the S-wave scattering length of K^-d scattering.

The real part of $f_0^{K^-d}(0)$ is related to $a_0^{K^-d}$ as

$$\mathcal{R}e f_0^{K^-d}(0) = a_0^{K^-d}. \quad (1.5)$$

Due to the optical theorem the imaginary part of the amplitude $f_0^{K^-d}(0)$ can be expressed in terms of the total cross-section $\sigma_0^{K^-d}(Q)$ for K^-d scattering in the S-wave state

$$\mathcal{I}m f_0^{K^-d}(0) = \lim_{Q\to 0}\frac{Q}{4\pi}\sigma_0^{K^-d}(Q) = \frac{1}{2}\lim_{Q\to 0}\frac{1}{Q}\big(1 - \eta_0^{K^-d}(Q)\cos 2\delta_0^{K^-d}(Q)\big). \quad (1.6)$$

The r.h.s. of (1.6) can be transcribed into the form

$$\mathcal{I}m f_0^{K^-d}(0) = -\frac{1}{2}\frac{\mathrm{d}\eta_0^{K^-d}(Q)}{\mathrm{d}Q}\bigg|_{Q=0}. \quad (1.7)$$

Hence, according to the DGBT formula the energy level displacement of the ground state of kaonic hydrogen is defined by

$$\epsilon_{1s} = -2\alpha^3\mu^2\mathcal{R}e f_0^{K^-d}(0) = -2\alpha^3\mu^2 a_0^{K^-d},$$
$$\Gamma_{1s} = 4\alpha^3\mu^2\mathcal{I}m f_0^{K^-d}(0) = -2\alpha^3\mu^2\frac{\mathrm{d}\eta_0^{K^-d}(Q)}{\mathrm{d}Q}\bigg|_{Q=0}. \quad (1.8)$$

These are general expressions describing the energy level displacement of the ground state of kaonic deuterium.

The paper is organized as follows. In sect. 2 we give the wave function of the ground state of kaonic deuterium in the momentum and the particle number representations. We derive the general formula for the energy level displacement of the ground state of kaonic deuterium in terms of the amplitude of K^-d scattering for arbitrary relative momenta of the K^-d pair and define the S-wave amplitude $f_0^{K^-d}(0)$ of elastic K^-d scattering in terms of the S-wave amplitudes of elastic K^-p, K^-n and $K^-(pn)_{^3\mathrm{S}_1}$ scattering, where the np pair couples in the $^3\mathrm{S}_1$ state with isospin zero. In sect. 3 we compute the S-wave amplitude of elastic K^-n scattering near threshold. In sect. 4 we derive the Ericson-Weise formula for the S-wave scattering length of K^-d scattering. In sect. 5 we adduce the general formula for the S-wave amplitude of elastic K^-pn scattering, saturated by the intermediate two-baryon states $NY = n\Lambda^0$, $n\Sigma^0$ and $p\Sigma^-$. In sects. 6, 7 and 8 we compute the amplitudes of the reactions $K^-(pn)_{^3\mathrm{S}_1} \to NY \to K^-(pn)_{^3\mathrm{S}_1}$ with the NY pair in the $^3\mathrm{P}_1$ and $^1\mathrm{P}_1$ states for $NY = n\Lambda^0$, $n\Sigma^0$ and $p\Sigma^-$ pairs, respectively. We compute the contribution of these reactions to the energy level displacement of the ground state of kaonic hydrogen. In sect. 9 we compare our results with experimental data on the rates of the reactions $K^-d \to n\Lambda^0$, $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$, other theoretical approaches and estimate the expected value of the total width and the shift of the ground state of kaonic deuterium, which are $\Gamma_{1s} = (630 \pm 100)\,\mathrm{eV}$ and $\epsilon_{1s} = (325 \pm 60)\,\mathrm{eV}$. In the conclusion we discuss the obtained result. For the details of calculations we relegate readers to [11].

2 Energy level displacement and the wave function of kaonic deuterium in the ground state

2.1 Energy level displacement of the ground state of kaonic deuterium. General formula

According to [4–6] the energy level displacement of the ground state of kaonic deuterium is defined by

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = \frac{1}{4m_K m_d}\frac{1}{3}\sum_{\lambda_d=0,\pm1}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{(2\pi)^3} \times\sqrt{\frac{m_K m_d}{E_K(\vec{k})E_d(\vec{k})}}\sqrt{\frac{m_K m_d}{E_K(\vec{q})E_d(\vec{q})}} \times\Phi^\dagger_{1s}(\vec{k})M(K^-(\vec{q})d(-\vec{q},\lambda_d) \to K^-(\vec{k})d(-\vec{k},\lambda_d))\Phi_{1s}(\vec{q}), \quad (2.1)$$

where we have averaged over polarizations of the deuteron $\lambda_d = 0, \pm1$, $\Phi^\dagger_{1s}(\vec{k})$ and $\Phi_{1s}(\vec{q})$ are the wave functions of kaonic deuterium in the ground state in the momentum representation and $M(K^-(\vec{q})d(-\vec{q},\lambda_d) \to K^-(\vec{k})d(-\vec{k},\lambda_d))$ is the amplitude of elastic K^-d scattering[1]. Due to the wave functions $\Phi^\dagger_{1s}(\vec{k})$ and $\Phi_{1s}(\vec{q})$ the main contributions to the integrals over $\vec{k}$ and $\vec{q}$ come from

[1] Of course, the energy level displacement of the ground state of kaonic deuterium does not depend on the polarization of the deuteron and formula (2.1) is valid for a fixed λ_d.

the regions of 3-momenta $k \sim 1/a_{\rm B}$ and $q \sim 1/a_{\rm B}$, where $1/a_{\rm B} \simeq \alpha\mu \simeq 3\,{\rm MeV}$. Since typical momenta in the integrand are much less than the masses of coupled particles, $m_d \gg m_K \gg 1/a_{\rm B}$, the amplitude of K^-d scattering can be defined for low-energy momenta only[2].

In the low-energy limit $k, q \to 0$ the relation (2.1) can be transcribed into the form [4–6]

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = \frac{2\pi}{\mu} f_0^{K^-d}(0)|\Psi_{1s}(0)|^2\Big(1 + \delta^{(\rm sm)}_{1s}\Big), \quad (2.2)$$

where $\delta^{(\rm sm)}_{1s}$ is equal to [4]

$$\delta^{(\rm sm)}_{1s} = -\alpha\frac{\mu}{m_K}\frac{8}{\sqrt{\pi}}\frac{\Gamma(3/4)}{\Gamma(1/4)} \simeq -10^{-2}. \quad (2.3)$$

The correction $\delta^{(\rm sm)}_{1s}$ is universal and related to the smearing of the wave function of exotic atom in the ground state around the origin $r = 0$ [4].

For the analysis of the energy level displacement of the ground state of kaonic deuterium in terms of the K^-N and K^-NN interactions we suggest the following. According to [4–6] the energy level displacement of the ground state of kaonic deuterium can be defined as

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = \lim_{T,V\to\infty}\frac{\Big\langle A^{(1s)}_{K^-d}(\vec{P},\lambda_d)|\mathbb{T}|A^{(1s)}_{K^-d}(\vec{P},\lambda_d)\Big\rangle}{2E^{(1s)}_A(\vec{P})VT}\Bigg|_{\vec{P}=0}, \quad (2.4)$$

where $E^{(1s)}_A(\vec{P}) = \sqrt{\vec{P}^2 + (M^{(1s)}_A)^2}$ is the total energy of kaonic deuterium in the ground state[3], TV is a 4-dimensional volume defined by $(2\pi)^4\delta^{(4)}(0) = TV$ [12] and $\mathbb{T}$ is the T-matrix obeying the unitary condition [12]

$$\mathbb{T} - \mathbb{T}^\dagger = i\mathbb{T}^\dagger\mathbb{T}. \quad (2.5)$$

Then, $|A^{(1s)}_{\pi d}(\vec{P},\lambda_d)\rangle$ is the ground-state wave function of kaonic deuterium in the momentum and particle number representation.

[2] It is obvious that due to formula (2.1) a knowledge of the amplitude of K^-d scattering for all relative momenta from zero to infinity should give a possibility to calculate the energy level displacement of the ground state of kaonic deuterium without any low-energy approximation.

[3] Here $M^{(1s)}_A = m_K + m_d + E_{1s}$ and $E_{1s} = -\alpha^2\mu/2 = -10.41\,{\rm keV}$ are the mass and the binding energy of kaonic deuterium in the ground state.

2.2 Wave function of kaonic deuterium in the ground state

The wave function $|A^{(1s)}_{\pi d}(\vec{P},\lambda_d)\rangle$ of kaonic deuterium in the ground state we determine as

$$\begin{aligned}
&\Big|A^{(1s)}_{K^-d}(\vec{P},\lambda_d = \pm1)\Big\rangle = \\
&\frac{1}{(2\pi)^3}\int\frac{{\rm d}^3k_K}{\sqrt{2E_K(\vec{k}_K)}}\frac{{\rm d}^3k_d}{\sqrt{2E_d(\vec{k}_d)}} \\
&\times\sqrt{2E^{(1s)}_A(\vec{k}_K + \vec{k}_d)}\,\delta^{(3)}(\vec{P} - \vec{k}_K - \vec{k}_d)\Phi_{1s}(\vec{k}_K) \\
&\times\frac{1}{(2\pi)^3}\int\frac{{\rm d}^3k_p}{\sqrt{2E_p(\vec{k}_p)}}\frac{{\rm d}^3k_n}{\sqrt{2E_n(\vec{k}_n)}} \\
&\times\sqrt{2E_d(\vec{k}_p + \vec{k}_n)}\,\delta^{(3)}(\vec{k}_d - \vec{k}_p - \vec{k}_n)\Phi_d\Big(\frac{\vec{k}_p - \vec{k}_n}{2}\Big) \\
&\times c^\dagger_{K^-}(\vec{k}_K)a^\dagger_p(\vec{k}_p,\pm1/2)a^\dagger_n(\vec{k}_n,\pm1/2)|0\rangle, \\
&\Big|A^{(1s)}_{K^-d}(\vec{P},\lambda_d = 0)\Big\rangle = \\
&\frac{1}{(2\pi)^3}\int\frac{{\rm d}^3k_K}{\sqrt{2E_K(\vec{k}_K)}}\frac{{\rm d}^3k_d}{\sqrt{2E_d(\vec{k}_d)}} \\
&\times\sqrt{2E^{(1s)}_A(\vec{k}_K + \vec{k}_d)}\,\delta^{(3)}(\vec{P} - \vec{k}_K - \vec{k}_d)\Phi_{1s}(\vec{k}_K) \\
&\times\frac{1}{(2\pi)^3}\int\frac{{\rm d}^3k_p}{\sqrt{2E_p(\vec{k}_p)}}\frac{{\rm d}^3k_n}{\sqrt{2E_n(\vec{k}_n)}} \\
&\times\sqrt{2E_d(\vec{k}_p + \vec{k}_n)}\,\delta^{(3)}(\vec{k}_d - \vec{k}_p - \vec{k}_n)\Phi_d\Big(\frac{\vec{k}_p - \vec{k}_n}{2}\Big) \\
&\times c^\dagger_{K^-}(\vec{k}_K)\frac{1}{\sqrt{2}}\Big[a^\dagger_p(\vec{k}_p,+1/2)a^\dagger_n(\vec{k}_n,-1/2) \\
&+ a^\dagger_p(\vec{k}_p,-1/2)a^\dagger_n(\vec{k}_n,+1/2)\Big]|0\rangle, \qquad (2.6)
\end{aligned}$$

where $\Phi_d(\vec{k})$ is the wave function of the deuteron as a bound np state with a total isospin zero, $I = 0$, and a total spin one, $S = 1$. It is normalized to unity [4]

$$\int\frac{{\rm d}^3k}{(2\pi)^3}|\Phi_d(\vec{k})|^2 = 1. \quad (2.7)$$

The operators $c^\dagger_{K^-}(\vec{k}_K)$, $a^\dagger_p(\vec{k}_p,\sigma_p)$ and $a^\dagger_n(\vec{k}_n,\sigma_n)$ create the K^--meson, the proton and the neutron and obey standard canonical and relativistic covariant commutation (for the K^--meson) and anti-commutation (for the proton and the neutron) relations. In appendix A of [11] we show that the wave function (2.6) describes the np pair in the bound $^3{\rm S}_1$ state with a total isospin zero, $I = 0$. One can show [4] that the wave function (2.6) is normalized as

$$\Big\langle A^{(1s)}_{\pi d}(\vec{P}',\lambda'_d)|A^{(1s)}_{\pi d}(\vec{P},\lambda_d)\Big\rangle = (2\pi)^3 2E^{(1s)}_A(\vec{P})\delta^{(3)}(\vec{P}' - \vec{P})\delta_{\lambda'_d\lambda_d}. \quad (2.8)$$

Using the wave function (2.6) the energy level displacement of the ground state of kaonic deuterium can be transcribed into the form

$$
\begin{aligned}
-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} &= \int \frac{d^3k\, d^3K}{(2\pi)^6} \frac{\Phi^*_{1s}(\vec{k})\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_K(\vec{k})2E_p(\vec{K})}} \\
&\times \int \frac{d^3q\, d^3Q}{(2\pi)^6} \frac{\Phi_{1s}(\vec{q})\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_K(\vec{q})2E_p(\vec{Q})}} \\
&\times (2\pi)^3\delta^{(3)}(\vec{K}+\vec{k}-\vec{Q}-\vec{q})M(K^-(\vec{q})p(\vec{Q},\sigma_p) \to \\
&K^-(\vec{k})p(\vec{K},\sigma_p)) \\
&+ \int \frac{d^3k\, d^3K}{(2\pi)^6} \frac{\Phi^*_{1s}(\vec{k})\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_K(\vec{k})2E_n(\vec{K})}} \\
&\times \int \frac{d^3q\, d^3Q}{(2\pi)^6} \frac{\Phi_{1s}(\vec{q})\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_K(\vec{q})2E_n(\vec{Q})}} \\
&\times (2\pi)^3\delta^{(3)}(\vec{K}+\vec{k}-\vec{Q}-\vec{q})M(K^-(\vec{q})n(\vec{Q},\sigma_n) \to \\
&K^-(\vec{k})n(\vec{K},\sigma_n)) \\
&+ \int \frac{d^3k\, d^3K}{(2\pi)^6} \frac{\Phi^*_{1s}(\vec{k})\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_K(\vec{k})2E_p(\vec{K})2E_n(\vec{K}+\vec{k})}} \\
&\times \int \frac{d^3q\, d^3Q}{(2\pi)^6} \frac{\Phi_{1s}(\vec{q})\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_K(\vec{q})2E_p(\vec{Q})2E_n(\vec{Q}+\vec{q})}} \\
&\times M(K^-(\vec{q})p(\vec{Q},\sigma_p)n(-\vec{Q}-\vec{q},\sigma_n) \to \\
&K^-(\vec{k})p(\vec{K},\sigma_p)n(-\vec{K}-\vec{k},\sigma_n)) \\
&+ \int \frac{d^3k\, d^3K}{(2\pi)^6} \frac{\Phi^*_{1s}(\vec{k})\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_p(\vec{K})2E_n(\vec{K}+\vec{k})}} \\
&\times \int \frac{d^3q\, d^3Q}{(2\pi)^6} \frac{\Phi_{1s}(\vec{q})\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_p(\vec{Q})2E_n(\vec{Q}+\vec{q})}} \\
&\times (2\pi)^3\delta^{(3)}(\vec{k}-\vec{q})M(p(\vec{Q},\sigma_p)n(-\vec{Q}-\vec{q},\sigma_n) \\
&\to p(\vec{K},\sigma_p)n(-\vec{K}-\vec{k},\sigma_n)). \qquad (2.9)
\end{aligned}
$$

The r.h.s. of (2.9) is expressed in terms of the amplitudes of the reactions $K^-p \to K^-p$, $K^-n \to K^-n$ and $K^-(np)_{^3S_1} \to K^-(np)_{^3S_1}$, where the np pair couples in the $^3S_1$ with isospin zero. In principle, the amplitudes of the reactions $K^-p \to K^-p$, $K^-n \to K^-n$ and $K^-(np)_{^3S_1} \to K^-(np)_{^3S_1}$ should contain all corrections caused by QCD isospin-breaking and electromagnetic interactions and all inelastic channels induced by both strong, QCD isospin-breaking and electromagnetic interactions.

We would like to accentuate that the contribution of the last term in (2.9), describing the transition $np \to np$, should be dropped, since it corresponds to a disconnected Feynman diagram of elastic low-energy K^-d scattering.

In order to show this we represent the amplitude of elastic low-energy K^-d scattering as

$$
\begin{aligned}
&M(K^-(\vec{q})d(-\vec{q},\lambda_d) \to K^-(\vec{k})d(-\vec{k},\lambda_d)) = \\
&\lim_{T,V\to\infty} \frac{\langle K^-(\vec{k})d(-\vec{k},\lambda_d)|\mathbb{T}|K^-(\vec{q})d(-\vec{q},\lambda_d)\rangle}{VT}. \qquad (2.10)
\end{aligned}
$$

The wave function of the state $|K^-(\vec{k}_K)d(\vec{k}_d,\lambda_d)\rangle$ we define as [12]

$$
\big|K^-(\vec{k}_K)d(\vec{k}_d,\lambda_d)\big\rangle = c^\dagger_{K^-}(\vec{k}_K)|d(\vec{k}_d,\lambda_d)\rangle, \qquad (2.11)
$$

where $|d(\vec{k}_d,\lambda_d)\rangle$ is the wave function of the deuteron in the ground state, which we take in the form [4–6]

$$
\begin{aligned}
\big|d(\vec{k}_d,\lambda_d=\pm1)\big\rangle &= \frac{1}{(2\pi)^3}\int \frac{d^3k_p}{\sqrt{2E_p(\vec{k}_p)}}\frac{d^3k_n}{\sqrt{2E_n(\vec{k}_n)}} \\
&\times\sqrt{2E_d(\vec{k}_p+\vec{k}_n)}\,\delta^{(3)}(\vec{k}_d-\vec{k}_p-\vec{k}_n) \\
&\times\Phi_d\Big(\frac{\vec{k}_p-\vec{k}_n}{2}\Big)a^\dagger_p(\vec{k}_p,\pm1/2)a^\dagger_n(\vec{k}_n,\pm1/2)|0\rangle, \\
\big|d(\vec{k}_d,\lambda_d=0)\big\rangle &= \frac{1}{(2\pi)^3}\int \frac{d^3k_p}{\sqrt{2E_p(\vec{k}_p)}}\frac{d^3k_n}{\sqrt{2E_n(\vec{k}_n)}} \\
&\times\sqrt{2E_d(\vec{k}_p+\vec{k}_n)}\,\delta^{(3)}(\vec{k}_d-\vec{k}_p-\vec{k}_n) \\
&\times\Phi_d\Big(\frac{\vec{k}_p-\vec{k}_n}{2}\Big)\frac{1}{\sqrt{2}}\Big[a^\dagger_p(\vec{k}_p,+1/2)a^\dagger_n(\vec{k}_n,-1/2) \\
&+a^\dagger_p(\vec{k}_p,-1/2)a^\dagger_n(\vec{k}_n,+1/2)\Big]|0\rangle, \qquad (2.12)
\end{aligned}
$$

normalized as

$$
\begin{aligned}
&\big\langle d(\vec{k}'_d,\lambda'_d)|d(\vec{k}_d,\lambda_d)\big\rangle = \\
&\qquad (2\pi)^3 2E_d(\vec{k}_d)\delta^{(3)}(\vec{k}'_d-\vec{k}_d)\delta_{\lambda'_d\lambda_d}. \qquad (2.13)
\end{aligned}
$$

Following the procedure expounded in appendix A of [11] of one can show that the wave function (2.12) describes the np pair in the bound $^3S_1$ state with isospin zero, $I=0$.

Using (2.12) for the calculation of the matrix elements of the T-matrix in (2.10), we get

$$
\begin{aligned}
&M(K^-(\vec{q})d(-\vec{q},\lambda_d) \to K^-(\vec{k})d(-\vec{k},\lambda_d)) = \\
&\sqrt{2E_d(\vec{k})2E_d(\vec{q})} \\
&\times\int\frac{d^3K}{(2\pi)^3}\frac{d^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_p(\vec{K})}}\frac{\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_p(\vec{Q})}} \\
&\times(2\pi)^3\delta^{(3)}(\vec{K}+\vec{k}-\vec{Q}-\vec{q})M(K^-(\vec{q})p(\vec{Q},\sigma_p) \to \\
&K^-(\vec{k})p(\vec{K},\sigma_p)) + \sqrt{2E_d(\vec{k})2E_d(\vec{q})} \\
&\times\int\frac{d^3K}{(2\pi)^3}\frac{d^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_n(\vec{K})}}\frac{\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_n(\vec{Q})}}
\end{aligned}
$$

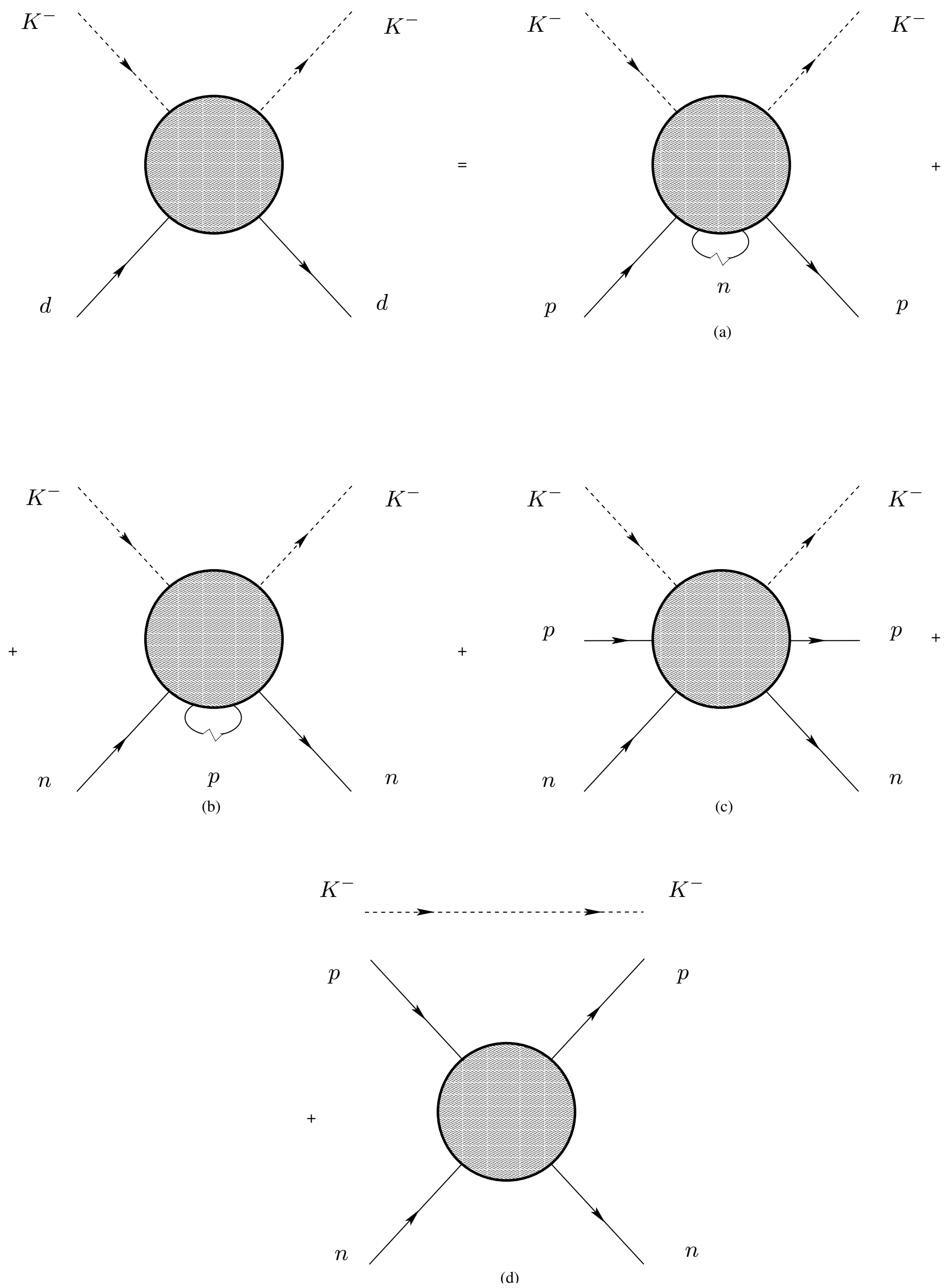

Fig. 1. Feynman diagrams of the reactions, describing the amplitude of low-energy elastic K^-d scattering.

$$\times(2\pi)^3\delta^{(3)}(\vec{K}+\vec{k}-\vec{Q}-\vec{q})M(K^-(\vec{q})n(\vec{Q},\sigma_n)\to K^-(\vec{k})n(\vec{K},\sigma_n))+\sqrt{2E_d(\vec{k})2E_d(\vec{q})}$$
$$\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_p(\vec{K})2E_n(\vec{k}+\vec{K})}}\times\frac{\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_p(\vec{Q})2E_n(\vec{q}+\vec{Q})}}$$
$$\times M(K^-(\vec{q})p(\vec{Q},\sigma_p)n(-\vec{q}-\vec{Q},\sigma_n)\to K^-(\vec{k})p(\vec{K},\sigma_p)n(-\vec{k}-\vec{K},\sigma_n))$$
$$+(2\pi)^3 2E_K(\vec{k})\delta^{(3)}(\vec{k}-\vec{q})2E_d(\vec{k})\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\Phi^*_d\Big(\vec{K}+\frac{1}{2}\vec{k}\Big)\Phi_d\Big(\vec{Q}+\frac{1}{2}\vec{k}\Big)$$
$$\times\frac{M(p(\vec{Q},\sigma_p)n(-\vec{k}-\vec{Q},\sigma_n)\to p(\vec{K},\sigma_p)n(-\vec{k}-\vec{K},\sigma_n))}{\sqrt{2E_p(\vec{K})2E_n(\vec{k})2E_p(\vec{Q})2E_n(\vec{k}+\vec{Q})}}. \quad (2.14)$$

The amplitude of low-energy elastic K^-d scattering (2.14) can be represented by Feynman diagrams depicted in fig. 1. It is seen that the last term is described by the disconnected diagram and, therefore, it does not contribute to the amplitude of K^-d scattering. Dropping this term the amplitude of low-energy elastic K^-d scattering reads

$$\begin{aligned}&M(K^-(\vec{q})d(-\vec{q},\lambda_d)\to K^-(\vec{k})d(-\vec{k},\lambda_d))=\\&\sqrt{2E_d(\vec{k})2E_d(\vec{q})}\\&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_p(\vec{K})}}\frac{\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_p(\vec{Q})}}\\&\times(2\pi)^3\delta^{(3)}(\vec{K}+\vec{k}-\vec{Q}-\vec{q})M(K^-(\vec{q})p(\vec{Q},\sigma_p)\to\\&K^-(\vec{k})p(\vec{K},\sigma_p))+\sqrt{2E_d(\vec{k})2E_d(\vec{q})}\\&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_n(\vec{K})}}\frac{\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_n(\vec{Q})}}\\&\times(2\pi)^3\delta^{(3)}(\vec{K}+\vec{k}-\vec{Q}-\vec{q})M(K^-(\vec{q})n(\vec{Q},\sigma_n)\to\\&K^-(\vec{k})n(\vec{K},\sigma_n))+\sqrt{2E_d(\vec{k})2E_d(\vec{q})}\\&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K}+\vec{k}/2)}{\sqrt{2E_p(\vec{K})2E_n(\vec{k}+\vec{K})}}\\&\times\frac{\Phi_d(\vec{Q}+\vec{q}/2)}{\sqrt{2E_p(\vec{Q})2E_n(\vec{q}+\vec{Q})}}\\&\times M(K^-(\vec{q})p(\vec{Q},\sigma_p)n(-\vec{q}-\vec{Q},\sigma_n)\to\\&K^-(\vec{k})p(\vec{K},\sigma_p)n(-\vec{k}-\vec{K},\sigma_n)).\qquad(2.15)\end{aligned}$$

Now we able to define the S-wave amplitude of elastic K^-d scattering near threshold.

2.3 S-wave amplitude of elastic K⁻d scattering near threshold

From (2.15) the S-wave amplitude of K^-d scattering near threshold can be defined by

$$\begin{aligned}f^{K^-d}_0(0)=&\frac{1}{8\pi}\frac{1}{1+m_K/m_d}\\&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{|\Phi_d(\vec{K})|^2}{E_p(\vec{K})}M(K^-(\vec{0})p(\vec{K},\sigma_p)\to\\&K^-(\vec{0})p(\vec{K},\sigma_p))+\frac{1}{8\pi}\frac{1}{1+m_K/m_d}\\&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{|\Phi_d(\vec{K})|^2}{E_n(\vec{K})}M(K^-(\vec{0})n(\vec{K},\sigma_n)\to\\&K^-(\vec{0})n(\vec{K},\sigma_n))+\frac{1}{16\pi}\frac{1}{1+m_K/m_d}\\&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{\Phi^*_d(\vec{K})}{\sqrt{E_p(\vec{K})E_n(\vec{K})}}\\&\times\frac{\Phi_d(\vec{Q})}{\sqrt{E_p(\vec{Q})E_n(\vec{Q})}}\\&\times\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to\\&K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)),\qquad(2.16)\end{aligned}$$

where we have averaged over the polarizations of the np pair in the ${}^3\mathrm{S}_1$ state.

In terms of the S-wave amplitudes $f^{K^-p}_0(K)$ and $f^{K^-n}_0(K)$ of K^-p and K^-n scattering, respectively, the S-wave amplitude $f^{K^-d}_0(0)$ reads

$$\begin{aligned}f^{K^-d}_0(0)=&\frac{1}{1+m_K/m_d}\int\frac{\mathrm{d}^3K}{(2\pi)^3}\Big(1+\frac{m_K}{E_p(\vec{K})}\Big)\\&\times f^{K^-p}_0(K)|\Phi_d(\vec{K})|^2\\&+\frac{1}{1+m_K/m_d}\int\frac{\mathrm{d}^3K}{(2\pi)^3}\Big(1+\frac{m_K}{E_n(\vec{K})}\Big)\\&\times f^{K^-n}_0(K)|\Phi_d(\vec{K})|^2\\&+\frac{1}{16\pi}\frac{1}{1+m_K/m_d}\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\\&\times\frac{\Phi^*_d(\vec{K})}{\sqrt{E_p(\vec{K})E_n(\vec{K})}}\frac{\Phi_d(\vec{Q})}{\sqrt{E_p(\vec{Q})E_n(\vec{Q})}}\\&\times\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to\\&K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)).\qquad(2.17)\end{aligned}$$

The real part $\mathcal{R}ef^{K^-d}_0(0)$ of the S-wave amplitude of K^-d scattering is defined by

$$\begin{aligned}\mathcal{R}ef^{K^-d}_0(0)=&\frac{1+m_K/m_N}{1+m_K/m_d}\\&\times\Big(\mathcal{R}ef^{K^-p}_0(0)+\mathcal{R}ef^{K^-n}_0(0)\Big)\\&+\mathcal{R}e\tilde{f}^{K^-d}_0(0),\qquad(2.18)\end{aligned}$$

where we have set $m_n=m_p=m_N$ and denoted

$$\begin{aligned}&\mathcal{R}e\tilde{f}^{K^-d}_0(0)=\\&\frac{1}{16\pi}\frac{1}{1+m_K/m_d}\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\\&\times\frac{\Phi^*_d(\vec{K})}{\sqrt{E_p(\vec{K})E_n(\vec{K})}}\frac{\Phi_d(\vec{Q})}{\sqrt{E_p(\vec{Q})E_n(\vec{Q})}}\\&\times\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}\mathcal{R}eM(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to\\&K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)).\qquad(2.19)\end{aligned}$$

The imaginary part $\mathcal{I}mf^{K^-d}_0(0)$ of the S-wave amplitude of K^-d scattering is determined by the imaginary of the amplitude $\tilde{f}^{K^-d}_0(0)$ only. This gives

$$\mathcal{I}m f^{K^-d}_0(0) = \frac{1}{16\pi}\frac{1}{1+m_K/m_d}\int\frac{d^3K}{(2\pi)^3}\frac{d^3Q}{(2\pi)^3} \times\frac{\Phi^*_d(\vec{K})}{\sqrt{E_p(\vec{K})E_n(\vec{K})}}\frac{\Phi_d(\vec{Q})}{\sqrt{E_p(\vec{Q})E_n(\vec{Q})}} \times\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}\mathcal{I}m M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)). \quad (2.20)$$

We accentuate that the decomposition of the real part of the S-wave amplitude of K^-d scattering, given by (2.18), agrees well with that suggested by Ericson and Weise for the S-wave scattering length of π^-d scattering [3] and by Barrett and Deloff for the S-wave scattering length of K^-d scattering [13].

Thus, for the calculation of the energy level displacement of the ground state of kaonic deuterium we have to compute the amplitudes of K^-p, K^-n and $K^-(pn)_{{}^3\mathrm{S}_1}$ scattering near thresholds, $f^{K^-p}_0(0)$, $f^{K^-n}_0(0)$ and $\tilde{f}^{K^-d}_0(0)$, respectively. Since the amplitude $f^{K^-p}_0(0)$ has been computed in [6], it is left to compute the real part of the amplitude $f^{K^-n}_0(0)$ and the amplitude $\tilde{f}^{K^-d}(0)$.

3 S-wave amplitude of K⁻n scattering near threshold

The calculation of the S-wave amplitude $f^{K^-n}_0(Q)$ of K^-n scattering near threshold we carry out following [6]. The amplitude of low-energy K^-n scattering we represent in the form

$$f^{K^-n}_0(Q) = \frac{1}{2iQ}\Big(\eta^{K^-n}_0(Q)e^{2i\delta^{K^-n}_0(Q)} - 1\Big) = \frac{1}{2iQ}\Big(e^{2i\delta^{K^-n}_{\mathrm{B}}(Q)} - 1\Big) + e^{2i\delta^{K^-n}_{\mathrm{B}}(Q)} f^{K^-n}_0(Q)_{\mathrm{R}}, \quad (3.1)$$

where $\eta^{K^-n}_0(Q)$ and $\delta^{K^-n}_0(Q)$ are the inelasticity and the phase shift of low-energy K^-n scattering, which we describe in terms of $\delta^{K^-n}_0(Q)_{\mathrm{B}}$, the phase shift of an elastic background of low-energy K^-n scattering, and $f^{K^-n}_0(Q)_{\mathrm{R}}$, the contribution of the resonances. In the low-energy limit $\delta^{K^-n}_0(Q)_{\mathrm{B}} = A^{K^-n}_{\mathrm{B}}Q$, where $A^{K^-n}_{\mathrm{B}}$ is a real parameter, and

$$f^{K^-n}_0(0) = A^{K^-n}_{\mathrm{B}} + f^{K^-n}_0(0)_{\mathrm{R}}, \quad (3.2)$$

Since the state $|K^-n\rangle$ has the isospin one, $I=1$, we assume [6] that $f^{K^-n}_0(Q)_{\mathrm{R}}$ is defined by the contribution of the $\Sigma(1750)$-resonance with isospin $I=1$ and strangeness $S=-1$, the component of the $SU(3)_{\mathrm{flavour}}$ octet [14][4]. The effective Lagrangian of the $\Sigma(1750)\bar{K}N$ interaction reads [6]

$$\mathcal{L}_{\Sigma^-_2 BP}(x) = f_2\bar{\Sigma}^-_2(x)(\Sigma^-(x)\pi^0(x) - \Sigma^0(x)\pi^-(x)) + \frac{g_2}{\sqrt{3}}\bar{\Sigma}^-_2(x)\Lambda^0(x)\pi^-(x) - \frac{1}{\sqrt{2}}(g_2 - f_2)\bar{\Sigma}^-_2(x)n(x)K^-(x) + \mathrm{h.c.}, \quad (3.3)$$

where f_2 and g_2 are the phenomenological coupling constants [6]. The value $f_2 = -g_2/3$ has been fixed from the experimental data on the cross-sections for inelastic reactions $K^-p\to\Sigma^{\pm}\pi^{\mp}$, $K^-p\to\Sigma^0\pi^0$ and $K^-p\to\Lambda^0\pi^0$. The value $g_2 = 1.123$ has been calculated from the fit of the width of the $\Sigma(1750)$-resonance equal to $\Gamma_{\Sigma_2} = 50\,\mathrm{MeV}$ for mass $m_{\Sigma_2} = 1750\,\mathrm{MeV}$ [14].

3.1 Real part of $f^{K^-n}_0(0)_{\mathrm{R}}$

According to [6] the real part $\mathcal{R}e f^{K^-n}_0(0)_{\mathrm{R}}$ of the amplitude $f^{K^-n}_0(0)_{\mathrm{R}}$ is equal to

$$\mathcal{R}e f^{K^-n}_0(0)_{\mathrm{R}} = \frac{1}{4\pi}\frac{\mu}{m_K}\frac{8}{9}\frac{g^2_2}{m_{\Sigma_2} - m_K - m_N} = 0.037\,\mathrm{fm}. \quad (3.4)$$

The numerical value is obtained for $m_K = 494\,\mathrm{MeV}$ and $m_N = 940\,\mathrm{MeV}$.

Now we should proceed to computing the contribution of the smooth elastic background of low-energy K^-n scattering.

3.2 Elastic background of low-energy K⁻n scattering

Using the results obtained in [6] one can show that the smooth elastic background $A^{K^-n}_{\mathrm{B}}$ of low-energy elastic K^-n scattering does not contain the contribution of exotic four-quark states $a_0(980)$ and $f_0(980)$ and, as has been pointed out in [6], can be fully determined within the soft-kaon theorem and current algebra approach [15–17]. The result reads [6]

$$A^{K^-n}_{\mathrm{B}} = \frac{1}{8\pi}\frac{1}{F^2_K}\frac{m_K m_N}{m_K + m_N} = (0.200\pm 0.024)\,\mathrm{fm}, \quad (3.5)$$

where $F_K = 113\,\mathrm{MeV}$ is the PCAC constant of the K-mesons [8] and $\pm 0.024\,\mathrm{fm}$ is an uncertainty of the current algebra approach [6].

As has been shown in [6] the smooth elastic background of low-energy K^-n scattering can be also defined by the lowest quark box-diagram depicted in fig. 2, calculated with the effective quark model with chiral

[4] For simplicity we denote $\Sigma(1750)$ as Σ^-_2.

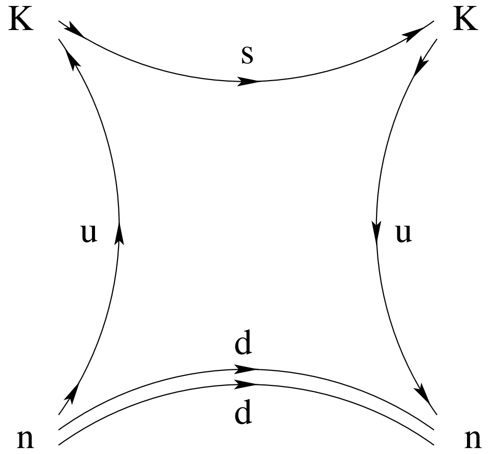

Fig. 2. The quark diagram describing a smooth elastic background of low-energy elastic K^-n scattering in the effective quark model with chiral $U(3)\times U(3)$ symmetry.

$U(3)\times U(3)$ symmetry [18–20]. Using the reduction technique [21] the amplitude of elastic low-energy K^-n scattering we define as

$$\begin{aligned}(2\pi)^4 i\delta^{(4)}(q'+p'-q-p)M(K^-n\to K^-n) = \\ \lim_{p'^2,p^2\to m_N^2,q'^2,q^2\to m_K^2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3\mathrm{d}^4x_4 \\ \times e^{iq'\cdot x_1+ip'\cdot x_2-ip\cdot x_3-iq\cdot x_4} \\ \times(\Box_1+m_K^2)(\Box_4+m_K^2)\bar u(p',\sigma'_n)\overrightarrow{(i\gamma_\nu\partial_2^\nu-m_N)} \\ \times\langle 0|\mathrm{T}(K^-(x_1)n(x_2)\bar n(x_3)K^+(x_4))|0\rangle \\ \times\overleftarrow{(-i\gamma_\mu\partial_3^\mu-m_N)}u(p,\sigma_n),\end{aligned} \quad (3.6)$$

where $n(x)$ and $u(p,\sigma_n)$ are the interpolating field operator and the Dirac bispinor of the neutron, and $K^{\pm}(x)$ are the interpolating fields of the $K^{\mp}$-mesons.

In order to describe the r.h.s. of eq. (3.6) at the quark level we follow [18] and use the equations of motion

$$\begin{aligned}\overrightarrow{(i\gamma_\nu\partial_2^\nu-m_N)}p(x_1) &= \frac{g_{\rm B}}{\sqrt2}\eta_n(x_2),\\ \bar p(x_3)\overleftarrow{(-i\gamma_\mu\partial_3^\mu-m_N)} &= \frac{g_{\rm B}}{\sqrt2}\bar\eta_n(x_3),\end{aligned} \quad (3.7)$$

where $\eta_n(x_2)$ and $\bar\eta_n(x_3)$ are the three-quark current densities [18]

$$\begin{aligned}\eta_n(x_2) &= -\varepsilon^{ijk}[\bar d^c{}_i(x_2)\gamma^\mu d_j(x_2)]\gamma_\mu\gamma^5 u_k(x_2),\\ \bar\eta_n(x_3) &= +\varepsilon^{ijk}\bar u_i(x_3)\gamma^\mu\gamma^5[\bar d_j(x_3)\gamma_\mu d^c_k(x_3)],\end{aligned} \quad (3.8)$$

where i, j and k are colour indices and $\bar\psi^c(x)=\psi(x)^T C$ and $C=-C^T=-C^\dagger=-C^{-1}$ is the charge conjugate matrix, T denotes transposition, and $g_{\rm B}$ is the phenomenological coupling constant of the low-lying baryon octet $B_8(x)$ coupled to the three-quark current densities [18]

$$\mathcal{L}_{\rm int}^{(\rm B)}(x) = \frac{g_{\rm B}}{\sqrt2}\bar B_8(x)\eta_8(x)+\text{h.c.} \quad (3.9)$$

The coupling constant $g_{\rm B}$ is equal to $g_{\rm B} = 1.34\times 10^{-4}\,\mathrm{MeV}^{-2}$ [18].

For the interpolating field operators of the $K^{\pm}$-mesons we use the following equations of motion [18]

$$\begin{aligned}(\Box_1+m_K^2)K^-(x_1) &= \frac{g_K}{\sqrt2}\bar u(x_1)i\gamma^5 s(x_1),\\ (\Box_4+m_K^2)K^+(x_4) &= \frac{g_K}{\sqrt2}\bar s(x_4)i\gamma^5 u(x_4),\end{aligned} \quad (3.10)$$

where $g_K=(m+m_s)/\sqrt2 F_K$, $m=330\,\mathrm{MeV}$ and $m_s=465\,\mathrm{MeV}$ are the masses of the constituent u, d and s quarks, respectively [18,20] (see also [22]).

The amplitude of low-energy elastic K^-p scattering is defined by

$$\begin{aligned}M(K^-n\to K^-n) &= -i\frac14 g_{\rm B}^2 g_K^2\\ &\times\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{d}^4x_3 e^{iq'\cdot x_1+ip'\cdot x_2-ip\cdot x_3}\bar u(p',\sigma'_n)\\ &\times\langle 0|\mathrm{T}(\bar u(x_1)i\gamma^5 s(x_1)\eta_n(x_2)\bar\eta_n(x_3)\bar s(0)i\gamma^5 u(0))|0\rangle\\ &\times u(p,\sigma_n),\end{aligned} \quad (3.11)$$

where the external momenta q', p', q and p should be kept on-mass shell $q'^2=q^2=m_K^2$ and $p'^2=p^2=m_N^2$.

In appendix B of [11] we have computed the vacuum expectation value. The parameter $A_{\rm B}^{K^-n}$, defining the smooth elastic background of low-energy K^-n scattering, is equal to

$$A_{\rm B}^{K^-n} = \frac{M(K^-n\to K^-n)}{8\pi(m_K+m_N)} = (0.221\pm0.024)\,\mathrm{fm}. \quad (3.12)$$

The value of the smooth elastic background of low-energy K^-n scattering, calculated with the effective quark model with chiral $U(3)\times U(3)$ symmetry, agrees well with that calculated within the soft-kaon theorem and current algebra approach (3.5).

3.3 Real part of the S-wave amplitude of K⁻n scattering near threshold and the S-wave scattering lengths of K⁻N scattering with isospin I = 0 and I = 1

Using the results obtained in sects. 3.1 and 3.2 we can compute the real part of the amplitude $f_0^{K^-n}(0)$ of K^-n scattering near threshold

$$\mathcal{R}e f_0^{K^-n}(0) = (0.258\pm0.024)\,\mathrm{fm}. \quad (3.13)$$

Since the K^-n couples in the state with isospin $I=1$, the real part of the S-wave amplitude $f_0^{K^-n}(0)$ defines the S-wave scattering length a_0^1 of $\bar KN$ scattering with isospin $I=1$: $a_0^1=(0.258\pm0.024)\,\mathrm{fm}$. Using the S-wave amplitude of K^-p scattering, calculated in [6], the S-wave scattering length a_0^0 of $\bar KN$ scattering is equal to: $a_0^0 = (-1.221\pm0.072)\,\mathrm{fm}$. The values

$$\begin{aligned}a_0^0 &= -1.221\pm0.072\,\mathrm{fm},\\ a_0^1 &= +0.258\pm0.024\,\mathrm{fm}\end{aligned} \quad (3.14)$$

we will use for the numerical calculation of the Ericson-Weise contribution [3] to the S-wave scattering length of K^-d scattering.

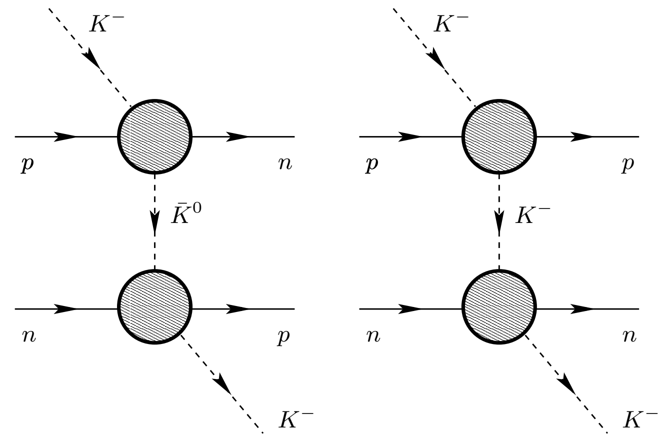

Fig. 3. Feynman diagrams of the amplitude of the low-energy reaction $K^-(pn)_{^3\mathrm{S}_1} \to K^-(pn)_{^3\mathrm{S}_1}$ defining the Ericson-Weise contribution to the S-wave scattering length of K^-d scattering.

4 Amplitude of $\mathrm{K^-(pn)_{^3S_1}}$ scattering near threshold

The amplitude of $K^-(pn)_{^3\mathrm{S}_1}$ scattering, $K^-(pn)_{^3\mathrm{S}_1} \to K^-(pn)_{^3\mathrm{S}_1}$, can be represented in the form of two main contributions: i) the amplitude, defining the S-wave scattering length of K^-d scattering in the Ericson-Weise form, described by the Feynman diagrams depicted in fig. 3 and caused by one-kaon exchanges, and ii) the amplitude, defined by the inelastic two-body $K^-(pn)_{^3\mathrm{S}_1} \to NY$ and three-body $K^-(pn)_{^3\mathrm{S}_1} \to NY\pi$ channels, where $Y = \Lambda^0, \Sigma^0$ or $\Sigma^{\pm}$ hyperons.

This gives the following analytical representation of the amplitude of the reaction $K^-(pn)_{^3\mathrm{S}_1} \to K^-(pn)_{^3\mathrm{S}_1}$:

$$\begin{aligned}&\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)} M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\ &\qquad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)) = \\ &\qquad \frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)} M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\ &\qquad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n))_{\mathrm{EW}} \\ &\qquad + \frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)} \tilde{M}(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\ &\qquad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)),\end{aligned} \tag{4.1}$$

where $M(K^-pn \to K^-pn)_{\mathrm{EW}}$ reproduces the Ericson-Weise formula of the S-wave scattering length of K^-d scattering, defined by the one-kaon exchanges, whereas $\tilde{M}(K^-pn \to K^-pn)$ is the amplitude of the reaction $K^-(pn)_{^3\mathrm{S}_1} \to K^-(pn)_{^3\mathrm{S}_1}$, saturated by the inelastic two-body $K^-(pn)_{^3\mathrm{S}_1} \to NY$ with $NY = n\Lambda^0, n\Sigma^0, p\Sigma^-$ and three-body $K^-(pn)_{^3\mathrm{S}_1} \to NY\pi$ with $NY\pi = n\Lambda^0\pi^0, p\Lambda^0\pi^-, n\Sigma^0\pi^0, n\Sigma^+\pi^-, p\Sigma^0\pi^-, p\Sigma^-\pi^0, n\Sigma^-\pi^+$ reactions.

4.1 The Ericson-Weise formula for $\mathrm{a_0^{K^-d}}$ scattering length

In the low-energy approximation the amplitude $M(K^-pn \to K^-pn)_{\mathrm{EW}}$ is defined in terms of the S-wave scattering lengths of $\bar{K}N$ scattering. For the calculation of $M(K^-pn \to K^-pn)_{\mathrm{EW}}$ with the np pair in the $^3\mathrm{S}_1$ state with isospin zero, $I = 0$, we suggest to use the following effective Lagrangian:

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}}(x) &= \bar{p}(x)(i\gamma^{\mu}\partial_{\mu} - m_N)p(x) \\ &+ \bar{n}(x)(i\gamma^{\mu}\partial_{\mu} - m_N)n(x) \\ &+ \partial_{\mu}K^{-\dagger}(x)\partial^{\mu}K^-(x) - m_K^2 K^{-\dagger}(x)K^-(x) \\ &+ \partial_{\mu}\bar{K}^{0\dagger}(x)\partial^{\mu}\bar{K}^0(x) - m_K^2\bar{K}^{0\dagger}(x)\bar{K}^0(x) \\ &+ 4\pi\Big(1 + \frac{m_K}{m_N}\Big)\Big[\frac{1}{2}\big(a_0^0 + a_0^1\big)K^{-\dagger}(x)K^-(x)\bar{p}(x)p(x) \\ &+ \frac{1}{2}\big(a_0^1 - a_0^0\big)\bar{K}^{0\dagger}(x)K^-(x)\bar{n}(x)p(x)\Big] \\ &+ 4\pi\Big(1 + \frac{m_K}{m_N}\Big)\Big[a_1 K^{-\dagger}(x)K^-(x)\bar{n}(x)n(x) \\ &+ \frac{1}{2}\big(a_0^1 - a_0^0\big)K^{-\dagger}(x)\bar{K}^0(x)\bar{p}(x)n(x)\Big],\end{aligned} \tag{4.2}$$

where a_0^0 and a_0^1 are the S-wave scattering lengths of $\bar{K}N$ scattering with isospin $I = 0$ and $I = 1$, respectively.

The effective action for the $K^-pn \to K^-pn$ transition in the one-kaon exchange approximation, described by the effective Lagrangian (4.2), reads

$$\begin{aligned}S_{\mathrm{eff}}^{(K^-pn)} = &-\int \mathrm{d}^4x\,\mathrm{d}^4y 4\pi^2\Big(1 + \frac{m_K}{m_N}\Big)^2 \\ &\times\Big[2a_0^1\big(a_0^0 + a_0^1\big)K^{-\dagger}(x)\bar{p}(x)p(x) \\ &\times\langle 0|\mathrm{T}(K^-(x)K^{-\dagger}(y))|0\rangle\bar{n}(y)n(y) \\ &+ \big(a_0^0 - a_0^1\big)^2 K^{-\dagger}(x)\bar{p}(x)n(x) \\ &\times\langle 0|\mathrm{T}(\bar{K}^0(x)\bar{K}^{0\dagger}(y))|0\rangle\bar{n}(y)p(y)\Big].\end{aligned} \tag{4.3}$$

Since in the case of isospin symmetry the vacuum expectation values of the $\bar{K}$-meson fields are equal

$$\begin{aligned}\langle 0|\mathrm{T}(K^-(x)K^{-\dagger}(y))|0\rangle &= \langle 0|\mathrm{T}(\bar{K}^0(x)\bar{K}^{0\dagger}(y))|0\rangle = \\ &-i\Delta(x-y) = \\ &\int\frac{\mathrm{d}^4k}{(2\pi)^4 i}\frac{e^{-ik\cdot(x-y)}}{m_K^2 - k^2 - i0},\end{aligned} \tag{4.4}$$

the r.h.s. of the effective action (4.3) can be transcribed into the form

$$\begin{aligned}S_{\mathrm{eff}}^{(K^-pn)} = &\, i\int \mathrm{d}^4x\,\mathrm{d}^4y 4\pi^2\Big(1 + \frac{m_K}{m_N}\Big)^2\Delta(x-y) \\ &\times\Big[2a_0^1\big(a_0^0 + a_0^1\big)K^{-\dagger}(x)K^-(y)\bar{p}(x)p(x)\bar{n}(y)n(y) \\ &+ \big(a_0^0 - a_0^1\big)^2 K^{-\dagger}(x)K^-(y)\bar{p}(x)n(x)\bar{n}(y)p(y)\Big].\end{aligned} \tag{4.5}$$

Now we have to take into account that the np pair couples in the $^3\mathrm{S}_1$ state with isospin zero, $I = 0$. This can be carried out by means of a Fierz transformation. Keeping only the term, describing the np pair in the $^3\mathrm{S}_1$ state with isospin zero, we get

$$\begin{aligned}
&\bar{p}(x)p(x)\bar{n}(y)n(y) \to \\
&\quad -\frac{1}{4}\bar{p}(x)\gamma_\mu n^c(y)\bar{n^c}(y)\gamma^\mu p(x) \\
&\quad +\frac{1}{8}\bar{p}(x)\sigma_{\mu\nu} n^c(y)\bar{n^c}(y)\sigma^{\mu\nu} p(x), \\
&\bar{p}(x)n(x)\bar{n}(y)p(y) \to \\
&\quad +\frac{1}{4}\bar{p}(x)\gamma_\mu n^c(y)\bar{n^c}(x)\gamma^\mu p(y) \\
&\quad -\frac{1}{8}\bar{p}(x)\sigma_{\mu\nu} n^c(y)\bar{n^c}(x)\sigma^{\mu\nu} p(y).
\end{aligned} \tag{4.6}$$

As has been shown in [23–26] the nucleon densities $\bar{p}(x)\gamma_\mu n^c(y)$ and $\bar{n^c}(x)\gamma^\mu p(y)$ have the quantum numbers of the deuteron. In the non-relativistic limit there survives only the $^3\mathrm{S}_1$ component of the np pair, whereas the $^3\mathrm{D}_1$ state is suppressed.

In order to understand the quantum numbers of the components of the tensor nucleon densities $\bar{p}(x)\sigma_{\mu\nu}n^c(y)$ and $\bar{n^c}(x)\sigma^{\mu\nu}p(y)$ it is convenient to represent the product of the tensor nucleon densities as follows:

$$\begin{aligned}
&\bar{p}(x)\sigma_{\mu\nu} n^c(y)\bar{n^c}(y)\sigma^{\mu\nu} p(x) = \\
&\quad -2\bar{p}(x)\gamma^0\vec{\gamma} n^c(y)\cdot \bar{n^c}(y)\gamma^0\vec{\gamma} p(x) \\
&\quad -2\bar{p}(x)\gamma^0\vec{\gamma}\gamma^5 n^c(y)\cdot \bar{n^c}(y)\gamma^0\vec{\gamma}\gamma^5 p(x), \\
&\bar{p}(x)\sigma_{\mu\nu} n^c(y)\bar{n^c}(x)\sigma^{\mu\nu} p(y) = \\
&\quad -2\bar{p}(x)\gamma^0\vec{\gamma} n^c(y)\cdot \bar{n^c}(x)\gamma^0\vec{\gamma} p(y) \\
&\quad -2\bar{p}(x)\gamma^0\vec{\gamma}\gamma^5 n^c(y)\cdot \bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5 p(y).
\end{aligned} \tag{4.7}$$

One can show that only nucleon densities $\bar{p}(x)\gamma^0\vec{\gamma}n^c(y)$ and $\bar{n^c}(y)\gamma^0\vec{\gamma}p(x)$ have the quantum numbers of the deuteron. However, the contribution of the $^3\mathrm{D}_1$ component enters with the sign opposite to that of the nucleon densities $\bar{p}(x)\gamma^\mu n^c(y)$ and $\bar{n^c}(x)\gamma^\mu p(y)$. Therefore, they coincide in the non-relativistic limits. Since the nucleon densities $\bar{p}(x)\gamma^0\vec{\gamma}\gamma^5 n^c(y)$ and $\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5 p(y)$ have not the quantum numbers of the deuteron, we will drop them from further consideration.

In the low-energy limit, when the masses of nucleons are much greater their relative 3-momenta, and using (4.7) we reduce the four-nucleon interaction in the effective action (4.6) to the form[5]

$$\begin{aligned}
&S_{\mathrm{eff}}^{(K^-(pn)_{^3\mathrm{S}_1})} = \\
&-i\int \mathrm{d}^4x\,\mathrm{d}^4y\, 2\pi^2\left(1+\frac{m_K}{m_N}\right)^2\big((a_0^0)^2 - 4a_0^0a_0^1 - (a_0^1)^2\big) \\
&\times \Delta(x-y)K^{-\dagger}(x)K^-(y)[\bar{p}(x)\vec{\gamma}n^c(x)]\cdot[\bar{n^c}(y)\vec{\gamma}p(y)].
\end{aligned} \tag{4.8}$$

[5] For the derivation of this effective action we have taken into account that in the non-relativistic limit $[\bar{p}(x)\gamma^0\vec{\gamma}n^c(y)]\cdot[\bar{n^c}(y)\gamma^0\vec{\gamma}p(x)] \to -[\bar{p}(x)\vec{\gamma}n^c(y)]\cdot[\bar{n^c}(y)\vec{\gamma}p(x)]$.

Using the effective action (4.8), we obtain the amplitude of the reaction $K^-(pn)_{^3\mathrm{S}_1} \to K^-(pn)_{^3\mathrm{S}_1}$, caused by the one-kaon exchanges, with the np pairs coupled in the $^3\mathrm{S}_1$ states with isospin zero:

$$\begin{aligned}
&M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\
&K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n))_{\mathrm{EW}} = \\
&-2\pi^2\left(1+\frac{m_K}{m_N}\right)^2\big((a_0^0)^2 - 4a_0^0a_0^1 - (a_0^1)^2\big) \\
&\times\frac{1}{m_K^2 + (\vec{K}-\vec{Q})^2} \\
&\times\big[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}n^c(-\vec{K},\sigma_n)\big]\cdot\big[\bar{u^c}(-\vec{Q},\sigma_n)\vec{\gamma}u(\vec{Q},\sigma_p)\big].
\end{aligned} \tag{4.9}$$

At low energies the summation over polarizations of the np pair in the $^3\mathrm{S}_1$ state gives (see appendix C of [11])

$$\begin{aligned}
&\frac{1}{3}\sum_{(\sigma_p,\sigma_n)} M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\
&\quad \pi^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n))_{\mathrm{EW}} = \\
&\quad 16\pi^2 m_N^2\left(1+\frac{m_K}{m_N}\right)^2\big((a_0^1)^2 + 4a_0^0a_0^1 - (a_0^0)^2\big) \\
&\quad \times\frac{1}{m_K^2 + (\vec{K}-\vec{Q})^2}.
\end{aligned} \tag{4.10}$$

Substituting (4.10) into (2.19), we get

$$\begin{aligned}
\tilde{f}_0^{K^-d}(0)_{\mathrm{EW}} &= \frac{1}{4}\left(1+\frac{m_K}{m_d}\right)^{-1}\left(1+\frac{m_K}{m_N}\right)^2 \\
&\times\big((a_0^1)^2 + 4a_0^0a_0^1 - (a_0^0)^2\big) \\
&\times\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\mathrm{d}^3Q}{(2\pi)^3}\frac{m_N}{E_N(\vec{K})}\Phi_d^*(\vec{K}) \\
&\times\frac{4\pi}{m_K^2 + (\vec{K}-\vec{Q})^2}\frac{m_N}{E_N(\vec{Q})}\Phi_d(\vec{Q}).
\end{aligned} \tag{4.11}$$

The expression (4.11) can be transcribed into the form suggested by Ericson and Weise [3]

$$\begin{aligned}
\tilde{f}_0^{K^-d}(0)_{\mathrm{EW}} &= \frac{1}{4}\left(1+\frac{m_K}{m_d}\right)^{-1}\left(1+\frac{m_K}{m_N}\right)^2 \\
&\times\big((a_0^1)^2 + 4a_0^0a_0^1 - (a_0^0)^2\big)\left\langle\frac{1}{r_{12}}\right\rangle,
\end{aligned} \tag{4.12}$$

where r_{12} is a distance between two scatterers n and p [3]. In our approach $\langle 1/r_{12}\rangle$ is defined by

$$\begin{aligned}
\left\langle\frac{1}{r_{12}}\right\rangle &= \int \mathrm{d}^3x\,\Psi_d^*(\vec{r})\frac{e^{-m_K r}}{r}\Psi_d(\vec{r}) = \\
&2\gamma_d E_1\left(\frac{m_N}{m_K + 2\gamma_d}\right) = 0.29\, m_\pi,
\end{aligned} \tag{4.13}$$

where $E_1(z)$ is the exponential integral [27], $\gamma_d = 1/r_d = 0.327\, m_\pi$ and $\Psi_d(\vec{r})$ is the wave function of the deuteron in the ground state in the coordinate representation. We have restricted the spatial region of the integration from below by the Compton wavelength of the nucleon [24].

The analogous calculation of the amplitude of $\pi^- d$ scattering [28] in the one-pion exchange approximation reproduces fully the Ericson-Weise formula [3]

$$\tilde{f}_0^{\pi^- d}(0)_{\rm EW} = 2\Big(1 + \frac{m_\pi}{m_d}\Big)^{-1}\Big(1 + \frac{m_\pi}{m_N}\Big)^2 (b_0^2 - 2b_1^2)\Big\langle \frac{1}{r_{12}}\Big\rangle, \quad (4.14)$$

where $b_0 = (a_0^{1/2} + 2a_0^{3/2})/3$ and $b_0 = (a_0^{3/2} - a_0^{1/2})/3$ are the isoscalar and isovector S-wave scattering lengths of πN scattering, $a_0^{1/2}$ and $a_0^{3/2}$ are the S-wave scattering lengths of πN scattering with isospin $I = 1/2$ and $I = 3/2$ and

$$\Big\langle \frac{1}{r_{12}}\Big\rangle = \int {\rm d}^3x \Psi_d^*(\vec{r}\,)\frac{e^{-m_\pi r}}{r}\Psi_d(\vec{r}\,) = 2\gamma_d E_1\Big(\frac{m_N}{m_\pi + 2\gamma_d}\Big) = 0.69\, m_\pi. \quad (4.15)$$

The numerical value (4.15) agrees well with the Ericson-Weise estimate $\langle 1/r_{12}\rangle = 0.64\, m_\pi$ [3].

Since the S-wave scattering lengths a_0^0 and a_0^1 of $\bar{K}N$ scattering are equal to $a_0^0 = (-1.221 \pm 0.072)$ fm and $a_0^1 = (0.258 \pm 0.024)$ fm (see (3.14)), the value of $\tilde{f}_0^{K^- d}(0)_{\rm EW}$ amounts to

$$\tilde{f}_0^{K^- d}(0)_{\rm EW} = (-0.254 \pm 0.021)\,{\rm fm}. \quad (4.16)$$

Thus, the S-wave scattering length of $K^- d$ scattering, defined by the Ericson-Weise formula, is equal to

$$\big(a_0^{K^- d}\big)_{\rm EW} = \frac{1 + m_K/m_N}{1 + m_K/m_d}\big(a_0^{K^- p} + a_0^{K^- n}\big) + \tilde{f}_0^{K^- d}(0)_{\rm EW} = (-0.525 \pm 0.094)\,{\rm fm}. \quad (4.17)$$

The total S-wave scattering length $a_0^{K^- d}$ of $K^- d$ scattering is defined by

$$a_0^{K^- d} = \big(a_0^{K^- d}\big)_{\rm EW} + \mathcal{R}e\tilde{f}_0^{K^- d}(0). \quad (4.18)$$

The S-wave amplitude $\tilde{f}_0^{K^- d}(0)$ of the reaction $K^-(pn)_{^3{\rm S}_1} \to K^-(pn)_{^3{\rm S}_1}$ is saturated by the inelastic two-body $K^-(pn)_{^3{\rm S}_1} \to NY$ and three-body $K^-(pn)_{^3{\rm S}_1} \to NY\pi$ channels

$$\tilde{f}_0^{K^- d}(0) = \tilde{f}_0^{K^- d}(0)_{(\text{two-body})} + \tilde{f}_0^{K^- d}(0)_{(\text{three-body})}, \quad (4.19)$$

where we have denoted

$$\tilde{f}_0^{K^- d}(0)_{(\text{two-body})} = \sum_{NY}\tilde{f}_0^{K^- d}(0)_{NY},$$
$$\tilde{f}_0^{K^- d}(0)_{(\text{three-body})} = \sum_{NY\pi}\tilde{f}_0^{K^- d}(0)_{NY\pi}. \quad (4.20)$$

The contribution of the reactions $K^-(pn)_{^3{\rm S}_1} \to N\Lambda^0\pi\pi$ can be neglected due to a smallness of the phase volume.

5 Inelastic two-body channels $K^-(pn)_{^3S_1} \to NY$. General formulas

The part of the width Γ_{1s} of the ground state of kaonic deuterium A_{Kd} is defined by the decays $A_{Kd} \to NY$, where $NY = n\Lambda^0, n\Sigma^0$ and $p\Sigma^-$. The other possible two-body decays as $A_{Kd} \to n\Lambda(1405)$ and $A_{Kd} \to N\Sigma(1385)$ are suppressed by the phase volume relative to the decays $A_{Kd} \to n\Lambda^0, n\Sigma^0$ and $p\Sigma^-$ [29–31].

The contribution of the decays $A_{Kd} \to n\Lambda^0, n\Sigma^0$ and $p\Sigma^-$ to the energy level displacement of the ground state of kaonic deuterium we take into account by computing the amplitude of the reaction $K^-(pn)_{^3{\rm S}_1} \to K^-(pn)_{^3{\rm S}_1}$, defined by the inelastic channels $K^-(pn)_{^3{\rm S}_1} \to NY \to K^-(pn)_{^3{\rm S}_1}$, where $NY = n\Lambda^0, n\Sigma^0$ and $p\Sigma^-$. At threshold in the reaction $K^-(pn)_{^3{\rm S}_1} \to NY$ the NY pair can be produced in the ${}^3{\rm P}_1$ and ${}^1{\rm P}_1$ state.

The amplitude of low-energy $K^-(pn)_{^3{\rm S}_1} \to K^-(pn)_{^3{\rm S}_1}$ scattering, caused by the contribution of two-body inelastic channels $K^-(pn)_{^3{\rm S}_1} \to NY \to K^-(pn)_{^3{\rm S}_1}$ with the NY pair coupled in the ${}^3{\rm P}_1$ and ${}^1{\rm P}_1$ state, we define as [4]

$$\begin{aligned}
&\tilde{M}(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\
&\quad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)) = \\
&\quad \int\frac{{\rm d}^3k_1}{(2\pi)^3 2E_n(\vec{k}_1)}\frac{{\rm d}^3k_2}{(2\pi)^3 2E_{\Lambda^0}(\vec{k}_2)} \\
&\quad \times\frac{(2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2)}{E_n(\vec{k}_1) + E_{\Lambda^0}(\vec{k}_2) - 2m_N - m_K - i0} \\
&\quad \times\sum_{(\alpha_2,\alpha_1;{}^3{\rm P}_1)} M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\
&\quad n(\vec{k}_1,\alpha_1)\Lambda^0(\vec{k}_2,\alpha_2);{}^3{\rm P}_1) \\
&\quad \times M(n(\vec{k}_1,\alpha_1)\Lambda^0(\vec{k}_2,\alpha_2) \to \\
&\quad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^3{\rm P}_1) \\
&\quad + \int\frac{{\rm d}^3k_1}{(2\pi)^3 2E_n(\vec{k}_1)}\frac{{\rm d}^3k_2}{(2\pi)^3 2E_{\Sigma^0}(\vec{k}_2)} \\
&\quad \times\frac{(2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2)}{E_n(\vec{k}_1) + E_{\Sigma^0}(\vec{k}_2) - 2m_N - m_K - i0} \\
&\quad \times\sum_{(\alpha_2,\alpha_1;{}^3{\rm P}_1)} M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to \\
&\quad n(\vec{k}_1,\alpha_1)\Sigma^0(\vec{k}_2,\alpha_2);{}^3{\rm P}_1) \\
&\quad \times M(n(\vec{k}_1,\alpha_1)\Sigma^0(\vec{k}_2,\alpha_2) \to \\
&\quad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^3{\rm P}_1) \\
&\quad + \int\frac{{\rm d}^3k_1}{(2\pi)^3 2E_p(\vec{k}_1)}\frac{{\rm d}^3k_2}{(2\pi)^3 2E_{\Sigma^-}(\vec{k}_2)} \\
&\quad \times\frac{(2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2)}{E_p(\vec{k}_1) + E_{\Sigma^-}(\vec{k}_2) - 2m_N - m_K - i0} \\
&\quad \times\sum_{(\alpha_2,\alpha_1;{}^3{\rm P}_1)} M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n) \to
\end{aligned}$$

$$p(\vec{k}_1,\alpha_1)\Sigma^-(\vec{k}_2,\alpha_2);{}^3\mathrm{P}_1)$$
$$\times M(p(\vec{k}_1,\alpha_1)\Sigma^-(\vec{k}_2,\alpha_2)\to$$
$$K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^3\mathrm{P}_1)$$
$$+\int\frac{\mathrm{d}^3k_1}{(2\pi)^3 2E_n(\vec{k}_1)}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2E_{\Lambda^0}(\vec{k}_2)}$$
$$\times\frac{(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)}{E_n(\vec{k}_1)+E_{\Lambda^0}(\vec{k}_2)-2m_N-m_K-i0}$$
$$\times\sum_{(\alpha_2,\alpha_1;{}^1\mathrm{P}_1)}M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to$$
$$n(\vec{k}_1,\alpha_1)\Lambda^0(\vec{k}_2,\alpha_2);{}^1\mathrm{P}_1)$$
$$\times M(n(\vec{k}_1,\alpha_1)\Lambda^0(\vec{k}_2,\alpha_2)\to$$
$$K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^1\mathrm{P}_1)$$
$$+\int\frac{\mathrm{d}^3k_1}{(2\pi)^3 2E_n(\vec{k}_1)}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2E_{\Sigma^0}(\vec{k}_2)}$$
$$\times\frac{(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)}{E_n(\vec{k}_1)+E_{\Sigma^0}(\vec{k}_2)-2m_N-m_K-i0}$$
$$\times\sum_{(\alpha_2,\alpha_1;{}^1\mathrm{P}_1)}M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to$$
$$n(\vec{k}_1,\alpha_1)\Sigma^0(\vec{k}_2,\alpha_2);{}^1\mathrm{P}_1)$$
$$\times M(n(\vec{k}_1,\alpha_1)\Sigma^0(\vec{k}_2,\alpha_2)\to$$
$$K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^1\mathrm{P}_1)$$
$$+\int\frac{\mathrm{d}^3k_1}{(2\pi)^3 2E_p(\vec{k}_1)}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2E_{\Sigma^-}(\vec{k}_2)}$$
$$\times\frac{(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)}{E_p(\vec{k}_1)+E_{\Sigma^-}(\vec{k}_2)-2m_N-m_K-i0}$$
$$\times\sum_{(\alpha_2,\alpha_1;{}^1\mathrm{P}_1)}M(K^-(\vec{0})p(\vec{Q},\sigma_p)n(-\vec{Q},\sigma_n)\to$$
$$p(\vec{k}_1,\alpha_1)\Sigma^-(\vec{k}_2,\alpha_2);{}^1\mathrm{P}_1)$$
$$\times M(p(\vec{k}_1,\alpha_1)\Sigma^-(\vec{k}_2,\alpha_2)\to$$
$$K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^1\mathrm{P}_1),\qquad(5.1)$$

where we have neglected the contribution of a kinetic energy of a relative motion of the np pair.

The real and imaginary parts of the S-wave amplitude $\tilde{f}_0^{K^-d}(0)$, caused by the intermediate states $(NY)_{{}^3\mathrm{P}_1}$ and $(NY)_{{}^1\mathrm{P}_1}$, we determine as

$$\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(NY;{}^3\mathrm{P}_1)}=$$
$$\frac{1}{512\pi^4}\frac{1}{1+m_K/m_d}\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}\sum_{(\alpha_2,\alpha_1;{}^3\mathrm{P}_1)}$$
$$\times\mathcal{P}\int\frac{\mathrm{d}^3k}{E_N(k)E_Y(k)}\frac{1}{E_N(\vec{k})+E_Y(\vec{k})-2m_N-m_K}$$
$$\times\Big|\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{E_N(\vec{K})}M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to$$
$$N(\vec{k},\alpha_1)Y(-\vec{k},\alpha_2);{}^3\mathrm{P}_1)\Big|^2,$$

$$\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(NY;{}^1\mathrm{P}_1)}=$$
$$\frac{1}{512\pi^4}\frac{1}{1+m_K/m_d}\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}\sum_{(\alpha_2,\alpha_1;{}^1\mathrm{P}_1)}$$
$$\times\mathcal{P}\int\frac{\mathrm{d}^3k}{E_N(k)E_Y(k)}\frac{1}{E_N(\vec{k})+E_Y(\vec{k})-2m_N-m_K}$$
$$\times\Big|\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{E_N(\vec{K})}M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to$$
$$N(\vec{k},\alpha_1)Y(-\vec{k},\alpha_2);{}^1\mathrm{P}_1)\Big|^2,\qquad(5.2)$$

where $\mathcal{P}$ means the principle value of the integral, and

$$\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(NY;{}^3\mathrm{P}_1)}=$$
$$\frac{1}{512\pi^3}\frac{1}{1+m_K/m_d}\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}\sum_{(\alpha_2,\alpha_1;{}^3\mathrm{P}_1)}$$
$$\times\int\frac{\mathrm{d}^3k}{E_N(k)E_Y(k)}\delta(E_N(\vec{k})+E_Y(\vec{k})-2m_N-m_K)$$
$$\times\Big|\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{E_N(\vec{K})}M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to$$
$$N(\vec{k},\alpha_1)Y(-\vec{k},\alpha_2);{}^3\mathrm{P}_1)\Big|^2,$$

$$\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(NY;{}^1\mathrm{P}_1)}=$$
$$\frac{1}{512\pi^3}\frac{1}{1+m_K/m_d}\frac{1}{3}\sum_{(\sigma_p,\sigma_n;{}^3\mathrm{S}_1)}\sum_{(\alpha_2,\alpha_1;{}^1\mathrm{P}_1)}$$
$$\times\int\frac{\mathrm{d}^3k}{E_N(k)E_Y(k)}\delta(E_N(\vec{k})+E_Y(\vec{k})-2m_N-m_K)$$
$$\times\Big|\int\frac{\mathrm{d}^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{E_N(\vec{K})}M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to$$
$$N(\vec{k},\alpha_1)Y(-\vec{k},\alpha_2);{}^1\mathrm{P}_1)\Big|^2.\qquad(5.3)$$

The amplitudes $M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to N(\vec{k},\alpha_1)Y(-\vec{k},\alpha_2))$ we suggest to compute within the approach developed in [19].

6 Amplitude of reaction $K^-(pn)_{{}^3\mathrm{S}_1}\to n\Lambda^0\to K^-(pn)_{{}^3\mathrm{S}_1}$ and the energy level displacement

The amplitudes of the reactions $K^-(pn)_{{}^3\mathrm{S}_1}\to n\Lambda^0$, where $n\Lambda^0$ pair is coupled in the ${}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ states, we define

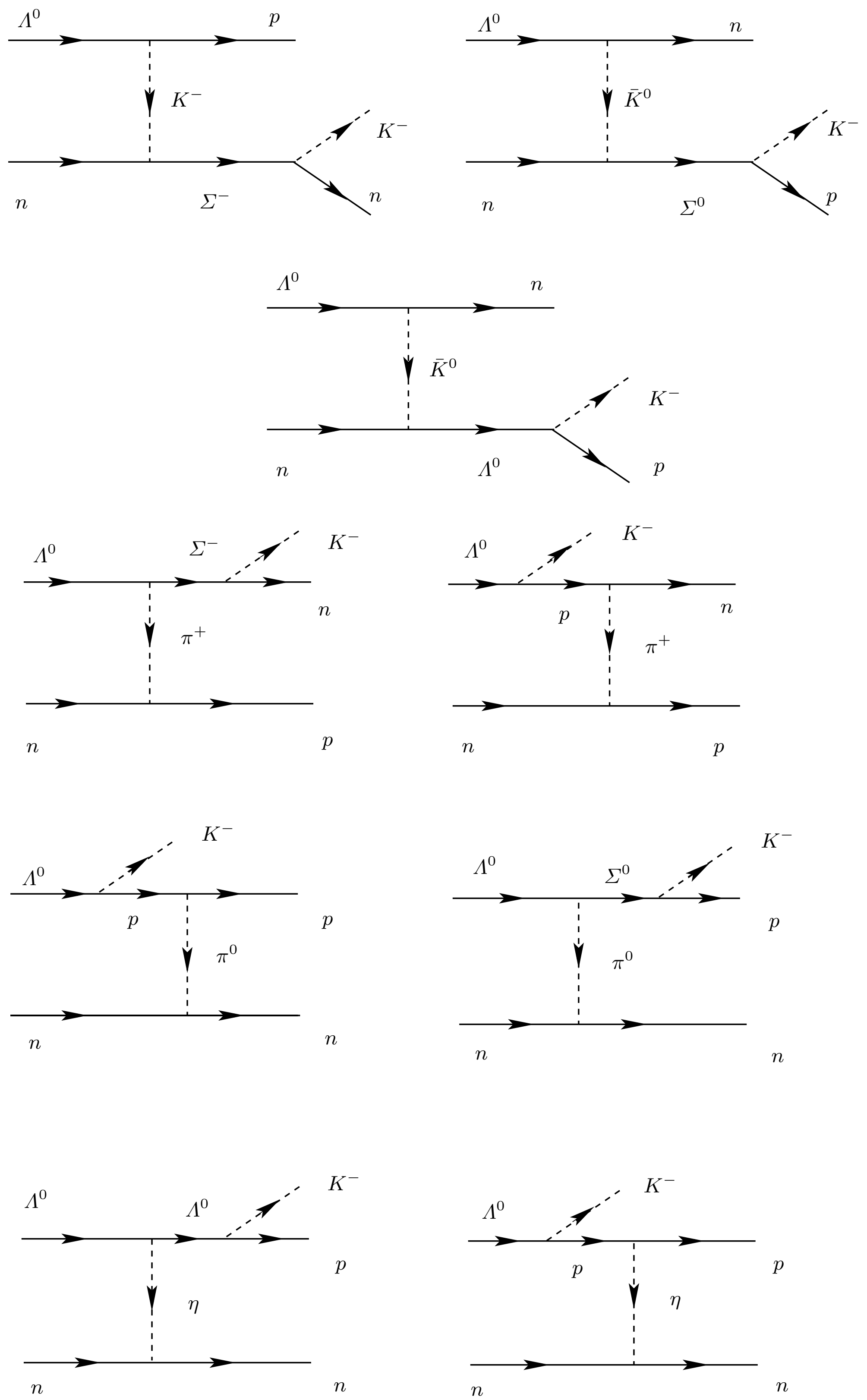

Fig. 4. Feynman diagrams describing the effective coupling constant of the transition $n\Lambda^0 \to K^- pn$ in the one-pseudoscalar-meson exchange approximation.

as

$$M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n) \to n(\vec{k},\alpha_1)\Lambda^0(-\vec{k},\alpha_2);{}^3\mathrm{P}_1) = -iC^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \times \frac{[\bar{u^c}(-\vec{K},\sigma_n)\vec{\gamma}u(\vec{K},\sigma_p)]\cdot[\bar{u}(-\vec{k},\alpha_2)\vec{\gamma}\gamma^5u^c(\vec{k},\alpha_1)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2+ia^t_{np}K} \times f^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0),$$

$$M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n) \to n(\vec{k},\alpha_1)\Lambda^0(-\vec{k},\alpha_2);{}^1\mathrm{P}_1) = -iC^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \times \frac{[\bar{u^c}(-\vec{K},\sigma_n)\vec{\gamma}u(\vec{K},\sigma_p)]\cdot[\bar{u}(-\vec{k},\alpha_2)\gamma^0\vec{\gamma}\gamma^5u^c(\vec{k},\alpha_1)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2+ia^t_{np}K} \times f^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0), \quad (6.1)$$

where $a^t_{np} = (5.424 \pm 0.004)\,\mathrm{fm} = (3.837 \pm 0.003)\,\mathrm{m}_\pi^{-1}$ and $r^t_{np} = (1.759 \pm 0.005)\,\mathrm{fm} = (1.244 \pm 0.004)\,\mathrm{m}_\pi^{-1}$ are the spin-triplet S-wave scattering length and effective range of np scattering in the ${}^3\mathrm{S}_1$ state [9]; $f^{(n\Lambda^0;X)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)$ is the amplitude of the final-state $n\Lambda^0$ interaction near threshold of the reaction $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Lambda^0)_X$ and $C^{(n\Lambda^0;X)}_{K^-(pn;{}^3\mathrm{S}_1)}$ is the effective coupling constant of the transition $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Lambda^0)_X$, where $X = {}^3\mathrm{P}_1$ or ${}^1\mathrm{P}_1$.

The spinorial wave functions of the $(np)_{{}^3\mathrm{S}_1}$, $(NY)_{{}^3\mathrm{P}_1}$ and $(NY)_{{}^1\mathrm{P}_1}$ states are analysed in appendix C of [11].

6.1 Effective coupling constant $\mathrm{C}^{n\Lambda^0}_{K^-pn}$

In the one-meson exchange approximation [19] the effective coupling constant of the transition $n\Lambda^0 \to K^-pn$ is defined by the Feynman diagrams are depicted in fig. 4. Following [19], the amplitude of the transition $n\Lambda^0 \to K^-pn$ [11], computed near threshold, we define by the local effective Lagrangian[6]

$$\mathcal{L}^{n\Lambda^0\to K^-pn}_{\mathrm{eff}}(x)_P =
-\frac{2}{\sqrt{3}}\frac{(3-2\alpha)(2\alpha-1)^2g^3_{\pi NN}}{m_K^2-(E_{\Lambda^0}-m_N)^2+k_0^2} \times \frac{1}{m_\Sigma+m_N+m_K}[\bar{p}(x)i\gamma^5\Lambda^0(x)][\bar{n}(x)n(x)]
-\frac{1}{\sqrt{3}}\frac{(3-2\alpha)(2\alpha-1)^2g^3_{\pi NN}}{m_K^2-(E_{\Lambda^0}-m_N)^2+k_0^2} \times \frac{1}{m_\Sigma+m_N+m_K}[\bar{n}(x)i\gamma^5\Lambda^0(x)][\bar{p}(x)n(x)]
+\frac{1}{3\sqrt{3}}\frac{(3-2\alpha)^3g^3_{\pi NN}}{m_K^2-(E_{\Lambda^0}-m_N)^2+k_0^2} \times \frac{1}{m_{\Lambda^0}+m_N+m_K}[\bar{n}(x)i\gamma^5\Lambda^0(x)][\bar{p}(x)n(x)]
-\frac{4}{\sqrt{3}}\frac{\alpha(2\alpha-1)g^3_{\pi NN}}{m_\pi^2-(E_N-m_N)^2+k_0^2} \times \frac{1}{m_\Sigma+m_N+m_K}[\bar{n}(x)\Lambda^0(x)][\bar{p}(x)i\gamma^5n(x)]
+\frac{2}{\sqrt{3}}\frac{(3-2\alpha)g^3_{\pi NN}}{m_\pi^2-(E_N-m_N)^2+k_0^2} \times \frac{m_N+m_K-m_{\Lambda^0}}{m_N^2-(E_{\Lambda^0}-m_K)^2+k_0^2} \times[\bar{n}(x)\Lambda^0(x)][\bar{p}(x)i\gamma^5n(x)]
-\frac{1}{\sqrt{3}}\frac{(3-2\alpha)g^3_{\pi NN}}{m_\pi^2-(E_N-m_N)^2+k_0^2} \times \frac{m_N+m_K-m_{\Lambda^0}}{m_N^2-(E_{\Lambda^0}-m_K)^2+k_0^2} \times[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)i\gamma^5n(x)]
+\frac{2}{\sqrt{3}}\frac{\alpha(2\alpha-1)g^3_{\pi NN}}{m_\pi^2-(E_N-m_N)^2+k_0^2} \times \frac{1}{m_\Sigma+m_N+m_K}[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)i\gamma^5n(x)]
-\frac{2}{3\sqrt{3}}\frac{\alpha(3-4\alpha)(3-2\alpha)g^3_{\pi NN}}{m_\eta^2-(E_N-m_N)^2+k_0^2} \times \frac{1}{m_{\Lambda^0}+m_N+m_K}[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)i\gamma^5n(x)]
+\frac{1}{3\sqrt{3}}\frac{(3-2\alpha)(3-4\alpha)^2g^3_{\pi NN}}{m_\eta^2-(E_N-m_N)^2+k_0^2} \times \frac{m_N+m_K-m_{\Lambda^0}}{m_N^2-(E_{\Lambda^0}-m_K)^2+k_0^2} \times[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)i\gamma^5n(x)], \quad (6.2)$$

where[7] $E_{\Lambda^0} = ((m_K+2m_N)^2+m^2_{\Lambda^0}-m^2_N)/2(m_K+2m_N) = 1262\,\mathrm{MeV}$, $E_N = ((m_K+2m_N)^2+m^2_N-m^2_{\Lambda^0})/2(m_K+2m_N) = 1108\,\mathrm{MeV}$ and $k_0 = ((2m_N+m_K)^2-(m_{\Lambda^0}+m_N)^2)^{1/2}((2m_N+m_K)^2-(m_{\Lambda^0}-m_N)^2)^{1/2}/2(2m_N+m_K) = 592\,\mathrm{MeV}$ are the energies and the relative momentum of the Λ^0-hyperon and the neutron at threshold. Then, for numerical calculations we set $g_{\pi NN} = 13.21$ [32] and $\alpha = 0.635$ [33] (see also [19]).

Now we have to take into account that the np pair couples in the ${}^3\mathrm{S}_1$ state with isospin zero. This can be carried out by means of Fierz transformation:

$$[\bar{p}(x)\gamma^5\Lambda^0(x)][\bar{n}(x)n(x)] \to -\frac{1}{4}[\bar{p}(x)\gamma_\mu n^c(x)][\bar{n^c}(x)\gamma^\mu\gamma^5\Lambda^0(x)] +\frac{1}{8}[\bar{p}(x)\sigma_{\mu\nu}n^c(x)][\bar{n^c}(x)\sigma^{\mu\nu}\gamma^5\Lambda^0(x)],$$

$$[\bar{n}(x)\gamma^5\Lambda^0(x)][\bar{p}(x)n(x)] \to +\frac{1}{4}[\bar{p}(x)\gamma_\mu n^c(x)][\bar{n^c}(x)\gamma^\mu\gamma^5\Lambda^0(x)] -\frac{1}{8}[\bar{p}(x)\sigma_{\mu\nu}n^c(x)][\bar{n^c}(x)\sigma^{\mu\nu}\gamma^5\Lambda^0(x)],$$

[6] The analytical expression of the Feynman diagrams in fig. 4 can be found in [11].

[7] The index P means that there are no scalar-meson exchange contributions.

$$\begin{aligned}
&[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)\gamma^5 n(x)] \to \\
&\quad +\frac{1}{4}[\bar{p}(x)\gamma_\mu n^c(x)][\bar{n^c}(x)\gamma^\mu\gamma^5\Lambda^0(x)] \\
&\quad +\frac{1}{8}[\bar{p}(x)\sigma_{\mu\nu} n^c(x)][\bar{n^c}(x)\sigma^{\mu\nu}\gamma^5\Lambda^0(x)], \\
&[\bar{n}(x)\Lambda^0(x)][\bar{p}(x)\gamma^5 n(x)] \to \\
&\quad -\frac{1}{4}[\bar{p}(x)\gamma_\mu n^c(x)][\bar{n^c}(x)\gamma^\mu\gamma^5\Lambda^0(x)] \\
&\quad -\frac{1}{8}[\bar{p}(x)\sigma_{\mu\nu} n^c(x)][\bar{n^c}(x)\sigma^{\mu\nu}\gamma^5\Lambda^0(x)]. \qquad (6.3)
\end{aligned}$$

In the non-relativistic limit the r.h.s. of these products can be reduced to the form

$$\begin{aligned}
&[\bar{p}(x)\gamma^5\Lambda^0(x)][\bar{n}(x)n(x)] \to \\
&\quad +\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)] \\
&\quad -\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)], \\
&[\bar{n}(x)\gamma^5\Lambda^0(x)][\bar{p}(x)n(x)] \to \\
&\quad -\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)] \\
&\quad +\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)], \\
&[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)\gamma^5 n(x)] \to \\
&\quad -\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)] \\
&\quad -\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)], \\
&[\bar{n}(x)\Lambda^0(x)][\bar{p}(x)\gamma^5 n(x)] \to \\
&\quad +\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)] \\
&\quad +\frac{1}{4}[\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)]. \qquad (6.4)
\end{aligned}$$

For the derivation of this Lagrangian we have taken into account that in the non-relativistic limit the four-baryon product reduces as follows:

$$\begin{aligned}
&[\bar{p}(x)\gamma^0\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)] \to \\
&\quad [\bar{p}(x)\vec{\gamma} n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)].
\end{aligned}$$

The effective low-energy four-baryon interactions $[\bar{p}(x)\vec{\gamma}n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)]$ and $[\bar{p}(x)\vec{\gamma}n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)]$ describe the transitions $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$ and $(n\Lambda^0)_{^1\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$, where the $n\Lambda^0$ pair couples in the $^3\mathrm{P}_1$ and $^1\mathrm{P}_1$ state, respectively.

6.2 Reaction $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$

The amplitude of the reaction $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$, where $n\Lambda^0$ pair couples to the np pair, which is in the $^3\mathrm{S}_1$ state, is defined by

$$\begin{aligned}
&M(n(\vec{k},\alpha_1)\Lambda^0(-\vec{k},\alpha_2) \to \\
&\quad K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^3\mathrm{P}_1) = iC^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \\
&\qquad \times \frac{[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)]\cdot[\bar{u^c}(\vec{k},\alpha_1)\vec{\gamma}\gamma^5 u(-\vec{k},\alpha_2)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2 - ia^t_{np}K} \\
&\qquad \times f^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0), \qquad (6.5)
\end{aligned}$$

where $f^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)$ is the amplitude, describing the $n\Lambda^0$ rescattering in the $^3\mathrm{P}_1$ state near threshold of the $K^-(pn)_{^3\mathrm{S}_1}$ system production and $C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$ is the effective coupling constant of the transition $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$.

Using (6.4) and (6.2) we obtain the effective Lagrangian of the transition $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$ near threshold:

$$\begin{aligned}
&\mathcal{L}^{(n\Lambda^0)_{^3\mathrm{P}_1}\to K^-(pn;{}^3\mathrm{S}_1)}_{\mathrm{eff}}(x)_P = \\
&\quad iC^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}K^{-\dagger}(x)[\bar{p}(x)\vec{\gamma}n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)], \qquad (6.6)
\end{aligned}$$

where we have denoted[8]

$$C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} = 1.7\times 10^{-6}\,\mathrm{MeV}^{-3}. \qquad (6.7)$$

The Lagrangian (6.6) describes the interaction of the $n\Lambda^0$ pair in the $^3\mathrm{P}_1$ state with the np pair in the $^3\mathrm{S}_1$ state through the emission of the K^--meson.

Using the results obtained in [6] one can show that the contribution of the resonances $\Lambda(1405)$ and $\Sigma(1750)$ to the effective coupling constant of the transition $n\Lambda^0 \to K^-pn$ is negligible small.

In fig. 5 we have depicted Feynman diagrams describing the contributions of the scalar mesons. The effective Lagrangian of the transition $n\Lambda^0 \to K^-pn$, caused by the scalar-meson exchange, is equal to

$$\begin{aligned}
\mathcal{L}^{n\Lambda^0}_{K^-pn}(x)_S = {}& \frac{1}{2\sqrt{3}}\frac{1}{g_AF_\pi}\frac{(3-2\alpha)g^2_{\pi NN}}{m^2_K-(E_{\Lambda^0}-m_N)^2+k^2_0} \\
&\times[\bar{p}(x)i\gamma^5\Lambda^0(x)][\bar{n}(x)n(x)] \\
&+\frac{1}{2\sqrt{3}}\frac{1}{g_AF_\pi}\frac{(3-2\alpha)g^2_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0} \\
&\times[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)i\gamma^5 n(x)] \\
&+\frac{1}{6\sqrt{3}}\frac{1}{g_AF_\pi}\frac{(3-2\alpha)(3-4\alpha)g^2_{\pi NN}}{m^2_\eta-(E_N-m_N)^2+k^2_0} \\
&\times[\bar{p}(x)\Lambda^0(x)][\bar{n}(x)i\gamma^5 n(x)] \\
&-\frac{1}{\sqrt{3}}\frac{1}{g_AF_\pi}\frac{(3-2\alpha)g^2_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0} \\
&\times[\bar{n}(x)\Lambda^0(x)][\bar{p}(x)i\gamma^5 n(x)]. \qquad (6.8)
\end{aligned}$$

[8] The analytical expression of $C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$ can be found in [11].

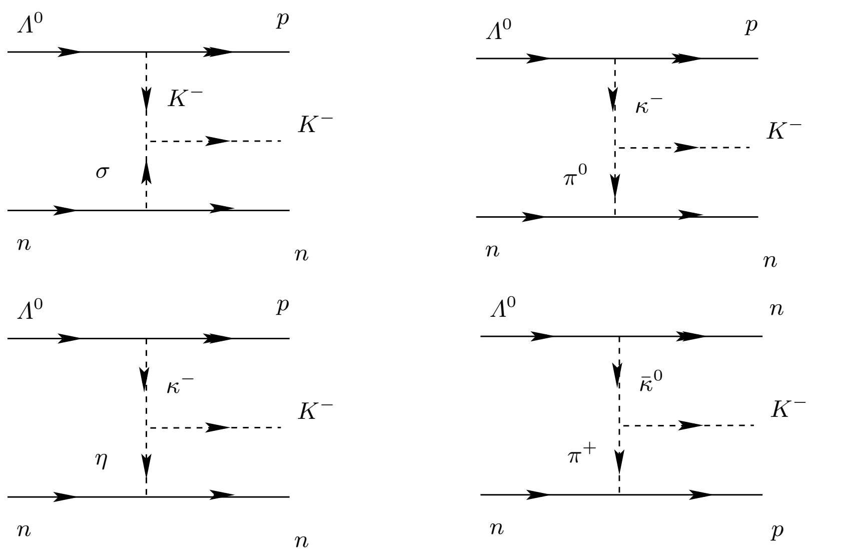

Fig. 5. Feynman diagrams describing the contribution of scalar mesons to the effective coupling constant of the transition $n\Lambda^0 \to K^- pn$ in the one-pseudoscalar-meson exchange approximation.

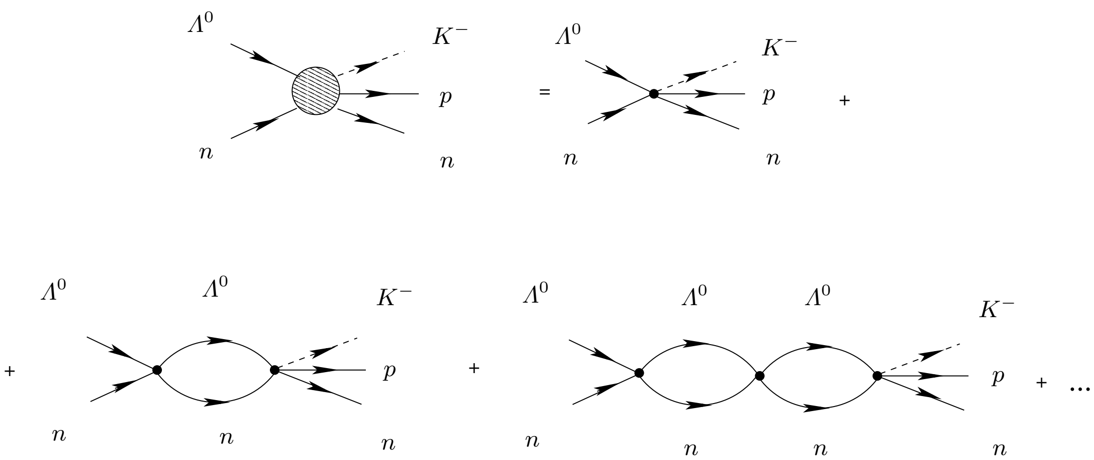

Fig. 6. Feynman diagrams describing the $(n\Lambda^0)_{^3\mathrm{P}_1}$ rescattering in the initial state of the reaction $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(np)_{^3\mathrm{S}_1}$.

The effective coupling constant of the $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$ transition, induced by the scalar-meson exchanges, reads[9]

$$\delta C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} = -1.3 \times 10^{-6}\,\mathrm{MeV}^{-3}. \tag{6.9}$$

Thus, the total effective coupling constant $C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$ of the $(n\Lambda^0)_{^3\mathrm{P}_1} \to K^-(pn)_{^3\mathrm{S}_1}$ transition, is equal to

$$C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} = 4 \times 10^{-7}\,\mathrm{MeV}^{-3}. \tag{6.10}$$

The contribution of the scalar mesons we have computed in the infinite mass limit. This corresponds to the non-linear realization of chiral symmetry [34] used within ChPT by Gasser and Leutwyler [35].

Now we proceed to computing the amplitude of the $n\Lambda^0$ rescattering in the $^3\mathrm{P}_1$ state.

[9] The analytical expression of $\delta C^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$ is given in [11].

6.3 Amplitude of $n\Lambda^0$ rescattering in the $^3\mathrm{P}_1$ state

The amplitude $f^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn)_{^3\mathrm{S}_1}}(k_0)$, describing the rescattering of the $n\Lambda^0$ pair in the $^3\mathrm{P}_1$ state near threshold of the $K^-(pn)_{^3\mathrm{S}_1}$ system production, is defined by the Feynman diagrams depicted in fig. 6[10]. The result of the calculation of these diagrams reads (see appendix E in [11] and [19])

$$\begin{aligned} &\left|f^{(n\Lambda^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\right| = \\ &\quad\left|\left\{1 - \frac{C_{n\Lambda^0}({}^3\mathrm{P}_1)}{12\pi^2}\frac{k_0^3}{E(k_0)}\left[\ln\left(\frac{E(k_0)+k_0}{E(k_0)-k_0}\right) - i\pi\right]\right\}^{-1}\right| \\ &\simeq 1, \end{aligned} \tag{6.11}$$

[10] Within the dispersion relation approach the final-state interaction of the baryon-baryon pairs (or baryon-baryon rescattering in the initial state) has been elaborated by Anisovich *et al.* [36].

where $E(k_0) = \sqrt{k_0^2 + m_B^2}$[11] and the effective coupling constant $C_{n\Lambda^0}(^3\mathrm{P}_1)$ is equal to [19]

$$C_{n\Lambda^0}(^3\mathrm{P}_1) = (3-2\alpha)^2 \frac{g^2_{\pi NN}}{12k_0^2}\ln\left(1 + \frac{4k_0^2}{m_K^2}\right) - \alpha(3-4\alpha)\frac{g^2_{\pi NN}}{6k_0^2}\ln\left(1+\frac{4k_0^2}{m_\eta^2}\right) = 2.0\times 10^{-4}\,\mathrm{MeV}^{-2}. \quad (6.12)$$

The rescattering of the $n\Lambda^0$ pair in the $^3\mathrm{P}_1$ state is defined by the interaction, computed in the one-meson exchange approximation (see [19]):

$$\mathcal{L}_{\rm eff}^{(n\Lambda^0)_{^3\mathrm{P}_1}\to(n\Lambda^0)_{^3\mathrm{P}_1}}(x) = -\frac{1}{4}C_{n\Lambda^0}(^3\mathrm{P}_1)[\bar{\Lambda}^0(x)\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)]. \quad (6.13)$$

For the derivation of the effective Lagrangian (6.13) we have used the relations

$$[\bar{\Lambda}^0(x)\gamma^5\Lambda^0(x)][\bar{n}(x)\gamma^5 n(x)] = +\frac{1}{4}[\bar{\Lambda}^0(x)\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)] + \frac{1}{4}[\bar{\Lambda}^0(x)\gamma^0\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)] + \cdots,$$
$$[\bar{\Lambda}^0(x)\gamma^5 n(x)][\bar{n}(x)\gamma^5\Lambda^0(x)] = +\frac{1}{4}[\bar{\Lambda}^0(x)\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Lambda^0(x)] + \frac{1}{4}[\bar{\Lambda}^0(x)\gamma^0\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)] + \cdots \quad (6.14)$$

caused by Fierz transformation.

6.4 S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}$ of K^-d scattering

For the calculation of the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}$ of K^-d scattering near threshold, caused by the exchange of the $n\Lambda^0$ pair in the $^3\mathrm{P}_1$ state, we have to square the spinorial wave functions of the coupled baryons and to sum over polarizations, taking into account that np and $n\Lambda^0$ pairs are in the $^3\mathrm{S}_1$ and $^3\mathrm{P}_1$ state, respectively. This yields (see appendix C of [11])

$$\frac{1}{3}\sum_{(\sigma_p,\sigma_n;^3\mathrm{S}_1)}\ \sum_{(\alpha_2,\alpha_1;^3\mathrm{P}_1)}\Big|\big[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)\big]\cdot\big[\bar{u^c}(\vec{k},\alpha_1)\vec{\gamma}\gamma^5 u(-\vec{k},\alpha_2)\big]\Big|^2 = \frac{128}{3}m_N^2\vec{k}^2. \quad (6.15)$$

First, let us compute the imaginary part of the amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}$. From (5.3) and taking into account the result of the summation over polarizations of baryons (6.15) we get

$$\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)} = \frac{1}{3\pi^2}\frac{1}{1+m_K/m_d}\times\frac{k_0^3}{2m_N+m_K}\Big[C^{(n\Lambda^0;^3\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}\Big]^2\Big|f^{(n\Lambda^0;^3\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}(k_0)\Big|^2 \times\Big|\int\frac{d^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2 - ia^t_{np}K}\Big|^2, \quad (6.16)$$

where we have integrated over $\vec{k}$. The integral over $\vec{K}$ can be calculated analytically and result reads

$$\int\frac{d^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2 - ia^t_{np}K} = \sqrt{\frac{2\gamma_d}{\pi^3}}\Big[\frac{\pi}{2}\frac{ab\gamma_d}{(a+\gamma_d)(b+\gamma_d)} + i\frac{ab}{b-a}\Big(\frac{b^2}{b^2-\gamma_d^2}\ln\frac{b}{\gamma_d} - \frac{a^2}{a^2-\gamma_d^2}\ln\frac{a}{\gamma_d}\Big)\Big] = (0.030 + i0.061)\,m_\pi^{3/2}, \quad (6.17)$$

where we have denoted

$$a = \frac{1}{r^t_{np}}\left(1 - \sqrt{1-\frac{2r^t_{np}}{a^t_{np}}}\right) = 0.327\,m_\pi,$$
$$b = \frac{1}{r^t_{np}}\left(1 + \sqrt{1-\frac{2r^t_{np}}{a^t_{np}}}\right) = 1.280\,m_\pi. \quad (6.18)$$

Thus, at threshold the imaginary part of the S-wave amplitude of K^-d scattering, caused by the two-body inelastic channel $K^-(pn)_{^3\mathrm{S}_1}\to(n\Lambda^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$, is equal to

$$\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)} = 4.6\times10^{-3}\frac{1}{3\pi^2}\frac{m_\pi^3}{1+m_K/m_d}\frac{k_0^3}{2m_N+m_K}\times\Big[C^{(n\Lambda^0;^3\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}\Big]^2\Big|f^{(n\Lambda^0;^3\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}(k_0)\Big|^2 = 0.9\times10^{-3}\,\mathrm{fm}. \quad (6.19)$$

The real part of $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}$ is defined by

$$\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)} = \frac{1}{256\pi^3}\frac{1}{1+m_K/m_d}\times\frac{1}{3}\sum_{(\sigma_p,\sigma_n;^3\mathrm{S}_1)}\ \sum_{(\alpha_1,\alpha_2;^3\mathrm{P}_1)}\frac{1}{2\pi}\mathcal{P}\int\frac{d^3k}{E_N(k)E_{\Lambda^0}(k)}\times\frac{1}{E_N(\vec{k})+E_{\Lambda^0}(\vec{k})-2m_N-m_K}\times\Big|\int\frac{d^3K}{(2\pi)^3}\frac{\Phi_d(\vec{K})}{E_N(\vec{K})}M(n(\vec{k},\alpha_1)\Lambda^0(-\vec{k},\alpha_2)\to K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^3\mathrm{P}_1)\Big|^2. \quad (6.20)$$

[11] For simplicity we use the equal masses of baryons for the calculation of the rescattering of the $n\Lambda^0$ pair, where $m_B = \sqrt{(2m_N+m_K)^2 - 4k_0^2}/2 = 1030\,\mathrm{MeV}$.

The real part of the integral over $\vec{k}$ is divergent. For the regularization of the divergent integral we introduce the cut-off Λ. Subtracting the divergent part and keeping the finite part dependent on the cut-off Λ the result of the integration over $\vec{k}$ reads[12]

$$\frac{1}{2\pi}\mathcal{P}\int\frac{d^3k}{E_N^2(\vec{k})}\frac{1}{E_N(\vec{k})+E_{\Lambda^0}(\vec{k})-2m_N-m_K-i0}=$$
$$\frac{\pi m_B}{2m_B+m_K}\Bigg\{\frac{2}{\pi}\arctan\Big(\frac{\Lambda}{m_B}\Big)-\frac{2}{\pi}\frac{k_0}{m_B}$$
$$\times\ln\left[\frac{\frac{\Lambda}{m_B+\sqrt{\Lambda^2+m_N^2}}+\frac{k_0}{m_B+\sqrt{k_0^2+m_B^2}}}{\frac{\Lambda}{m_B+\sqrt{\Lambda^2+m_B^2}}-\frac{k_0}{m_B+\sqrt{k_0^2+m_B^2}}}\right]\Bigg\}=$$
$$\frac{\pi}{2}\frac{1}{1+m_K/2m_B}F\Big(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\Big), \qquad (6.21)$$

where $m_B=\sqrt{(2m_N+m_K)^2-4k_0^2}/2=1030\,\text{MeV}$. For numerical analysis we set $\Lambda=m_N$[13].

Hence, at threshold the part of the S-wave amplitude of K^-d scattering, caused by the two-body inelastic channel $K^-(pn)_{^3\mathrm{S}_1}\to(n\Lambda^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$, reads

$$\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}=4.6\times10^{-3}\frac{1}{12\pi^2}$$
$$\times\frac{m_\pi^3}{1+m_K/m_d}\frac{k_0^2}{1+m_K/2m_B}\Big[C^{(n\Lambda^0)_{^3\mathrm{P}_1}}_{K^-(pn;^3\mathrm{S}_1)}\Big]^2$$
$$\times\Big|f^{(n\Lambda^0)_{^3\mathrm{P}_1}}_{K^-(pn;^3\mathrm{S}_1)}(k_0)\Big|^2F\Big(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\Big)=$$
$$-0.1\times10^{-3}\,\text{fm}. \qquad (6.22)$$

The S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}$ of K^-d scattering, caused by the two-body inelastic channel $K^-(pn)_{^3\mathrm{S}_1}\to(n\Lambda^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$, results in

$$\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}=(-0.1+i0.9)\times10^{-3}\,\text{fm}. \qquad (6.23)$$

Now we proceed to computing the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^1\mathrm{P}_1)}$ of K^-d scattering near threshold, saturated by the intermediate $(n\Lambda^0)_{^1\mathrm{P}_1}$ state.

[12] We assume also that after the subtraction of the divergent part the integrand is a peaked function around the point $E_N(\vec{k})+E_{\Lambda^0}(\vec{k})-2m_N-m_K=0$, *i.e.* around $|\vec{k}|=k_0$. That is valid for the imaginary part of the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;^3\mathrm{P}_1)}$. Due to this assumption we can take away the squared amplitude of the reaction $K^-(pn)_{^3\mathrm{S}_1}\to(n\Lambda^0)_{^3\mathrm{P}_1}$ at $|\vec{k}|=k_0$.

[13] We assume that the cut-off $\Lambda=m_N$ is an universal cut-off for the analysis of low-energy interactions of the deuteron near threshold. We relegate the readers to sect. 4, the analysis of the Ericson-Weise formula for the S-wave scattering length of K^-d scattering, and [24,28].

6.5 Reaction $(n\Lambda^0)_{^1\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$

The amplitude of the reaction $(n\Lambda^0)_{^1\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$ is defined by

$$M(n(\vec{k},\alpha_1)\Lambda^0(-\vec{k},\alpha_2)\to$$
$$K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^1\mathrm{P}_1)=$$
$$iC^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}$$
$$\times\frac{[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)]\cdot[\bar{u^c}(\vec{k},\alpha_1)\vec{\gamma}\gamma^5u(-\vec{k},\alpha_2)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2+ia^t_{np}K}$$
$$\times f^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}(k_0), \qquad (6.24)$$

where $f^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}(k_0)$ is the amplitude, describing the $n\Lambda^0$ rescattering near threshold of the $K^-(pn)_{^3\mathrm{S}_1}$ system production, and $C^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}$ is the effective coupling constant of the transition $(n\Lambda^0)_{^1\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$.

The effective Lagrangian of the transition $(n\Lambda^0)_{^1\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$ at threshold can be defined by

$$\mathcal{L}_{\text{eff}}^{(n\Lambda^0;^1\mathrm{P}_1)\to K^-(pn;^3\mathrm{S}_1)}(x)=iC^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}$$
$$\times K^{-\dagger}(x)[\bar{p}(x)\vec{\gamma}n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)]. \qquad (6.25)$$

The effective coupling constant $C^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}$ is equal to[14]

$$C^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}=6\times10^{-7}\,\text{MeV}^{-3}. \qquad (6.26)$$

The effective coupling constant (6.26) contains the contribution of the scalar-meson exchanges computed in the infinite mass limit.

The Lagrangian (6.25) describes the interaction of the $n\Lambda^0$ pair in the $^1\mathrm{P}_1$ state with the np pair in the $^3\mathrm{S}_1$ through the emission of the K^--meson.

6.6 Amplitude of $(n\Lambda^0)_{^1\mathrm{P}_1}$ rescattering

The amplitude $f^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-pn}(k_0)$, describing the rescattering of the $n\Lambda^0$ pair in the $^1\mathrm{P}_1$ state near threshold of the $K^-(pn)_{^3\mathrm{S}_1}$ system production, is defined by the Feynman diagrams analogous to those depicted in fig. 6. The result of the calculation of these diagrams reads (see appendix E in [11] and [19])

$$\Big|f^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}(k_0)\Big|=$$
$$\left|\left\{1-\frac{C_{n\Lambda^0}(^1\mathrm{P}_1)}{24\pi^2}\frac{k_0^3}{E(k_0)}\Big[\ln\Big(\frac{E(k_0)+k_0}{E(k_0)-k_0}\Big)-i\pi\Big]\right\}^{-1}\right|$$
$$\simeq1, \qquad (6.27)$$

[14] The analytical expression of $C^{(n\Lambda^0;^1\mathrm{P}_1)}_{K^-(pn;^3\mathrm{S}_1)}$ is given in [11].

where $E(k_0) = \sqrt{k_0^2 + m_B^2}$[15] and the effective coupling constant $C_{(n\Lambda^0)}({}^1\mathrm{P}_1)$ is equal to [19]

$$\begin{aligned}C_{n\Lambda^0}({}^1\mathrm{P}_1) &= (3-2\alpha)^2\frac{g^2_{\pi NN}}{12k_0^2}\ln\left(1+\frac{4k_0^2}{m_K^2}\right)\\ &\quad -\alpha(3-4\alpha)\frac{g^2_{\pi NN}}{6k_0^2}\ln\left(1+\frac{4k_0^2}{m_\eta^2}\right) = \\ &\quad 2.0\times 10^{-4}\,\mathrm{MeV}^{-2}. \end{aligned} \tag{6.28}$$

The rescattering of the $n\Lambda^0$ pair in the ${}^1\mathrm{P}_1$ state is defined by the interaction, computed in the one-meson exchange approximation (see [19]):

$$\begin{aligned}&\mathcal{L}_{\rm eff}^{(n\Lambda^0;{}^1\mathrm{P}_1)\to(n\Lambda^0;{}^1\mathrm{P}_1)}(x) = \\ &\quad -\frac{1}{4}C_{n\Lambda^0}({}^1\mathrm{P}_1)[\bar{\Lambda}^0(x)\gamma^0\vec{\gamma}\gamma^5 n^c(x)]\\ &\quad \cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Lambda^0(x)]. \end{aligned} \tag{6.29}$$

Now we can proceed to computing the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)}$ of K^-d scattering near threshold, saturated by the intermediate $(n\Lambda^0)_{{}^1\mathrm{P}_1}$ state.

6.7 S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)}$ of K^-d scattering

For the calculation of the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)}$ of K^-d scattering near threshold, caused by the inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to(n\Lambda^0)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$, we have to square the amplitude (6.24) and to sum over polarizations of baryons, taking into account that np and $n\Lambda^0$ pairs are in the ${}^3\mathrm{S}_1$ and ${}^1\mathrm{P}_1$ state, respectively. This yields (see appendix C of [11])

$$\begin{aligned}&\frac{1}{3}\sum_{(\sigma_p,\sigma_n)}\sum_{(\alpha_2,\alpha_1;{}^1\mathrm{P}_1)}\Big|\big[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)\big]\\ &\quad\cdot\big[\bar{u^c}(\vec{k},\alpha_1)\gamma^0\vec{\gamma}\gamma^5 u(-\vec{k},\alpha_2)\big]\Big|^2 = \frac{64}{3}m_N^2\vec{k}^2. \end{aligned} \tag{6.30}$$

At threshold the imaginary part of the S-wave amplitude of K^-d scattering, caused by the two-body inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to(n\Lambda^0)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$, is equal to

$$\begin{aligned}&\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)} = \\ &\quad 4.6\times10^{-3}\frac{1}{6\pi^2}\frac{m_\pi^3}{1+m_K/m_d}\frac{k_0^3}{2m_N+m_K}\\ &\quad\times\Big[C^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}\Big]^2\Big|f^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\Big|^2 = \\ &\quad 1.0\times10^{-3}\,\mathrm{fm}. \end{aligned} \tag{6.31}$$

[15] For simplicity we use the equal masses of baryons for the calculation of the rescattering of the $n\Lambda^0$ pair, where $m_B = \sqrt{(2m_N+m_K)^2-4k_0^2}/2 = 1030\,\mathrm{MeV}$.

The real part of $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)}$ reads

$$\begin{aligned}&\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)} = 4.6\times10^{-3}\frac{1}{24\pi^2}\frac{m_\pi^3}{1+m_K/m_d}\\ &\quad\times\frac{k_0^2}{1+m_K/2m_B}\Big[C^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}\Big]^2\\ &\quad\times\Big|f^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\Big|^2 F\left(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\right) = \\ &\quad -0.1\times10^{-3}\,\mathrm{fm}. \end{aligned} \tag{6.32}$$

Thus, for the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0)_{{}^1\mathrm{P}_1}}$ we get

$$\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)} = (-0.1+i1.0)\times10^{-3}\,\mathrm{fm}. \tag{6.33}$$

Now we can estimate the contribution of the two-body inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to n\Lambda^0\to K^-(pn)_{{}^3\mathrm{S}_1}$ to the S-wave amplitude $f_0^{K^-d}(0)_{n\Lambda^0}$ of K^-d scattering near threshold and the energy level displacement of the ground state of kaonic deuterium.

6.8 S-wave amplitude $f_0^{K^-d}(0)_{n\Lambda^0}$ and the energy level displacement

The S-wave amplitude of K^-d scattering saturated at threshold by the intermediate $n\Lambda^0$ state in the ${}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ is equal to the sum of the contributions (6.23) and (6.33)

$$\tilde{f}_0^{K^-d}(0)_{n\Lambda^0} = (-0.2+i1.9)\times10^{-3}\,\mathrm{fm}. \tag{6.34}$$

The contribution of the decay $A_{Kd}\to n\Lambda^0$ to the energy level displacement of the ground state of kaonic deuterium amounts to

$$\begin{aligned}-\epsilon_{1s}^{(n\Lambda^0)} + i\frac{\Gamma_{1s}^{(n\Lambda^0)}}{2} &= 602\tilde{f}_0^{K^-d}(0)_{n\Lambda^0}\\ &= (-0.1+i1.1)\,\mathrm{eV}. \end{aligned} \tag{6.35}$$

Hence, the partial width of the decay $A_{Kd}\to n\Lambda^0$ is equal to $\Gamma_{1s}^{(n\Lambda^0)} = 2.2\,\mathrm{eV}$.

According to [31], the experimental rate of the production of the $n\Lambda^0$ pair at threshold of the reaction $K^-d\to n\Lambda^0$ is equal to $R(K^-d\to n\Lambda^0) = (0.387\pm0.041)\%$.

Using our estimate of the partial width, $\Gamma_{1s}^{(n\Lambda^0)} = 2.2\,\mathrm{eV}$, and the experimental rate, $R(K^-d\to n\Lambda^0) = (0.387\pm0.041)\%$, we can estimate the expected value of the total width of the energy level of the ground state of kaonic deuterium

$$\Gamma_{1s} = \frac{\Gamma_{1s}^{(n\Lambda^0)}}{(0.387\pm0.041)\times10^{-2}} = (570\pm130)\,\mathrm{eV}. \tag{6.36}$$

We have taken into account the theoretical accuracy of the energy level displacement, which is about 20%: $-\epsilon_{1s}^{(n\Lambda^0)} + i\Gamma_{1s}^{(n\Lambda^0)}/2 = (-0.10\pm0.02)+i(1.10\pm0.22)\,\mathrm{eV}$.

Our expected value of the total width of the ground state of kaonic deuterium is by a factor of 2 smaller compared with the value of the total width predicted by Barrett and Deloff [13].

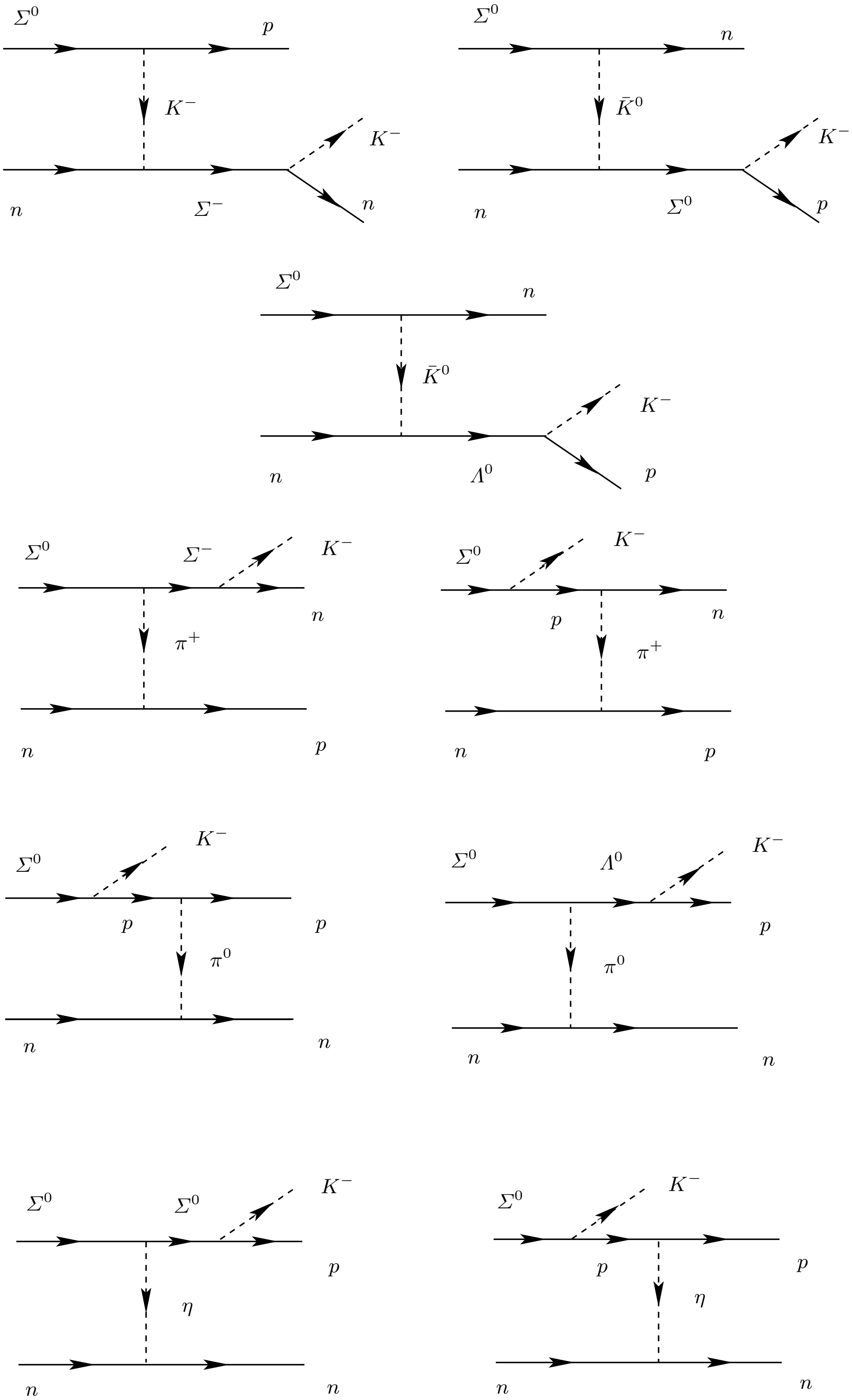

Fig. 7. Feynman diagrams describing the effective coupling constant of the transition $n\Sigma^0 \to K^- pn$ in the one-pseudoscalar-meson exchange approximation.

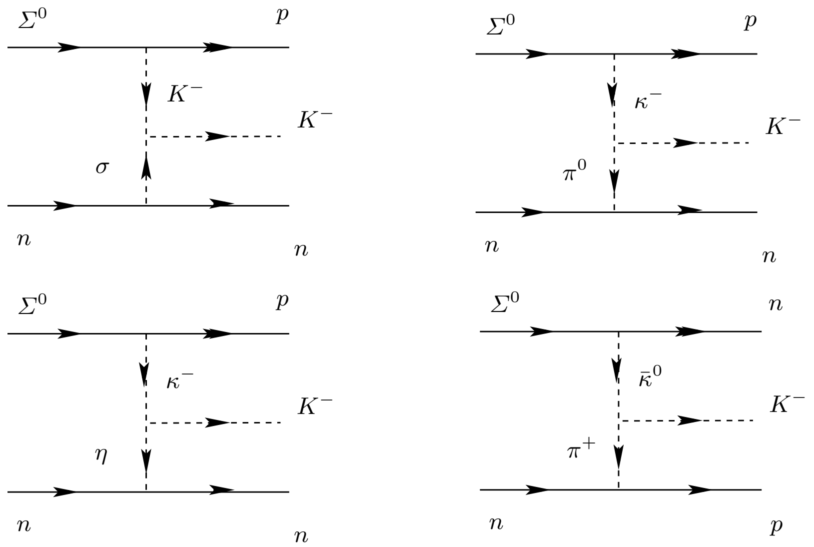

Fig. 8. Feynman diagrams describing the contribution of scalar mesons to the effective coupling constant of the transition $n\Sigma^0 \to K^- pn$ in the one-pseudoscalar-meson exchange approximation.

7 Amplitude of reaction $\mathbf{K^-(pn)_{{}^3S_1} \to n\Sigma^0 \to K^-(pn)_{{}^3S_1}}$ and the energy level displacement

The amplitudes of the reactions $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Sigma^0)_X$, where $n\Sigma^0$ pair couples in the $X = {}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ state, we define as

$$
\begin{aligned}
&M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n) \to \\
&\quad n(\vec{k},\alpha_1)\Sigma^0(-\vec{k},\alpha_2);{}^3\mathrm{P}_1) = -iC^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \\
&\quad\times\frac{\big[\bar{u^c}(-\vec{K},\sigma_n)\vec{\gamma}u(\vec{K},\sigma_p)\big]\cdot\big[\bar{u}(-\vec{k},\alpha_2)\vec{\gamma}\gamma^5u^c(\vec{k},\alpha_1)\big]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2+ia^t_{np}K} \\
&\quad\times f^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0), \\
&M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n) \to \\
&\quad n(\vec{k},\alpha_1)\Sigma^0(-\vec{k},\alpha_2);{}^1\mathrm{P}_1) = -iC^{(n\Sigma^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \\
&\quad\times\frac{\big[\bar{u^c}(-\vec{K},\sigma_n)\vec{\gamma}u(\vec{K},\sigma_p)\big]\cdot\big[\bar{u}(-\vec{k},\alpha_2)\gamma^0\vec{\gamma}\gamma^5u^c(\vec{k},\alpha_1)\big]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2+ia^t_{np}K} \\
&\quad\times f^{(n\Sigma^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0),
\end{aligned}
\tag{7.1}
$$

where $f^{(n\Sigma^0;X)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)$ is the amplitude of the final-state $n\Sigma^0$ interaction near threshold of the reaction $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Sigma^0)_X$ and $C^{(n\Sigma^0;X)}_{K^-(pn;{}^3\mathrm{S}_1)}$ is the effective coupling constant of the transition $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Sigma^0)_X$, where $X = {}^3\mathrm{P}_1$ or ${}^1\mathrm{P}_1$.

7.1 Effective coupling constant $\mathbf{C^{n\Sigma^0}_{K^-pn}}$

In the one-meson exchange approximation [19] the effective coupling constant $C^{n\Sigma^0}_{K^-pn}$ of the transition $n\Sigma^0 \to K^-pn$ is defined by the Feynman diagrams depicted in fig. 7. The analytical expression of the amplitude of the transition $n\Sigma^0 \to K^-pn$, determined by the Feynman diagrams in fig. 7 is given in [11].

The Feynman diagrams, describing the transition $n\Sigma^0 \to K^-pn$ through the scalar-meson exchanges, are depicted in fig. 8.

Taking the amplitude of the transition $n\Sigma^0 \to K^-pn$ at threshold, we can represent it in the form of the effective local Lagrangian

$$
\begin{aligned}
\mathcal{L}^{n\Sigma^0\to K^-pn}_{\mathrm{eff}}(x)_P &= \frac{2(2\alpha-1)^3g^3_{\pi NN}}{m^2_K-(E_{\Sigma^0}-m_N)^2+k^2_0} \\
&\times\frac{1}{m_\Sigma+m_N+m_K}[\bar{p}(x)i\gamma^5\Sigma^0(x)][\bar{n}(x)n(x)] \\
&-\frac{(2\alpha-1)^3g^3_{\pi NN}}{m^2_K-(E_{\Sigma^0}-m_N)^2+k^2_0}\,\frac{1}{m_\Sigma+m_N+m_K} \\
&\times[\bar{n}(x)i\gamma^5\Sigma^0(x)][\bar{p}(x)n(x)] \\
&+\frac{1}{3}\,\frac{(2\alpha-1)(3-2\alpha)^2g^3_{\pi NN}}{m^2_K-(E_{\Sigma^0}-m_N)^2+k^2_0}\,\frac{1}{m_{\Lambda^0}+m_N+m_K} \\
&\times[\bar{n}(x)i\gamma^5\Sigma^0(x)][\bar{p}(x)n(x)] \\
&-\frac{4(1-\alpha)(2\alpha-1)g^3_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0}\,\frac{1}{m_\Sigma+m_N+m_K} \\
&\times[\bar{n}(x)\Sigma^0(x)][\bar{p}(x)i\gamma^5n(x)] \\
&-\frac{2(2\alpha-1)g^3_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0}\,\frac{m_N+m_K-m_\Sigma}{m^2_N-(E_{\Sigma^0}-m_K)^2+k^2_0} \\
&\times[\bar{n}(x)\Sigma^0(x)][\bar{p}(x)i\gamma^5n(x)] \\
&+\frac{(2\alpha-1)g^3_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0}\,\frac{m_N+m_K-m_\Sigma}{m^2_N-(E_{\Sigma^0}-m_K)^2+k^2_0} \\
&\times[\bar{p}(x)\Sigma^0(x)][\bar{n}(x)i\gamma^5n(x)] \\
&-\frac{2}{3}\,\frac{\alpha(3-2\alpha)g^3_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0}\,\frac{m_N+m_K-m_\Sigma}{m^2_N-(E_{\Sigma^0}-m_K)^2+k^2_0}
\end{aligned}
$$

$$\times[\bar p(x)\Sigma^0(x)][\bar n(x)i\gamma^5 n(x)] - \frac{2}{3}\frac{\alpha(2\alpha-1)(3-2\alpha)g^3_{\pi NN}}{m^2_\eta-(E_N-m_N)^2+k_0^2}\frac{1}{m_\Sigma+m_N+m_K} \times[\bar p(x)\Sigma^0(x)][\bar n(x)i\gamma^5 n(x)] - \frac{1}{3}\frac{(3-4\alpha)^2 g^3_{\pi NN}}{m^2_\eta-(E_N-m_N)^2+k_0^2}\frac{m_N+m_K-m_\Sigma}{m^2_N-(E_{\Sigma^0}-m_K)^2+k_0^2} \times[\bar p(x)\Sigma^0(x)][\bar n(x)i\gamma^5 n(x)]. \quad (7.2)$$

The effective Lagrangian of the transition $n\Lambda^0 \to K^- pn$, caused by the scalar-meson exchanges, is equal to

$$\mathcal{L}^{n\Sigma^0}_{K^-pn}(x)_S = -\frac{1}{2}\frac{1}{g_A F_\pi}\frac{(2\alpha-1)g^2_{\pi NN}}{m^2_K-(E_{\Sigma^0}-m_N)^2+k_0^2} \times[\bar p(x)i\gamma^5\Sigma^0(x)][\bar n(x)n(x)] - \frac{1}{2}\frac{F_K}{g_A F^2_\pi}\frac{(2\alpha-1)g^2_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k_0^2} \times[\bar p(x)\Sigma^0(x)][\bar n(x)i\gamma^5 n(x)] - \frac{1}{6}\frac{F_K}{g_A F^2_\pi}\frac{(2\alpha-1)(3-4\alpha)g^2_{\pi NN}}{m^2_\eta-(E_N-m_N)^2+k_0^2} \times[\bar p(x)\Sigma^0(x)][\bar n(x)i\gamma^5 n(x)] - \frac{F_K}{g_A F^2_\pi}\frac{(2\alpha-1)g^2_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k_0^2} \times[\bar n(x)\Sigma^0(x)][\bar p(x)i\gamma^5 n(x)]. \quad (7.3)$$

Making the Fierz transformation (see (6.3)) we reduce the four-baryon operators to the form

$$[\bar p(x)\gamma^5\Sigma^0(x)][\bar n(x)n(x)] \to +\frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\vec\gamma\gamma^5\Sigma^0(x)] - \frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec\gamma\gamma^5\Sigma^0(x)],$$
$$[\bar n(x)\gamma^5\Sigma^0(x)][\bar p(x)n(x)] \to -\frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\vec\gamma\gamma^5\Sigma^0(x)] + \frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec\gamma\gamma^5\Sigma^0(x)],$$
$$[\bar p(x)\Sigma^0(x)][\bar n(x)\gamma^5 n(x)] \to -\frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\vec\gamma\gamma^5\Sigma^0(x)] - \frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec\gamma\gamma^5\Sigma^0(x)],$$
$$[\bar n(x)\Sigma^0(x)][\bar p(x)\gamma^5 n(x)] \to +\frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\vec\gamma\gamma^5\Sigma^0(x)] + \frac{1}{4}[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\gamma^0\vec\gamma\gamma^5\Sigma^0(x)]. \quad (7.4)$$

These relations make a projection of the four-baryon states onto the ${}^3\mathrm{S}_1\otimes{}^3\mathrm{P}_1$ and ${}^3\mathrm{S}_1\otimes{}^1\mathrm{P}_1$ states. Taking into account the relations (7.4) we can extract from the effective Lagrangian (7.2) the effective interactions, responsible for the transitions $(n\Sigma^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$ and $(n\Sigma^0)_{^1\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$ with the $n\Sigma^0$ pair in the ${}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ state.

7.2 Reaction $(\mathbf{n\Sigma^0})_{^3\mathrm{P}_1}\to \mathbf{K^-(pn)}_{^3\mathrm{S}_1}$

The amplitude of the reaction $(n\Sigma^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$, where the $n\Sigma^0$ pair in the ${}^3\mathrm{P}_1$ state couples to the np pair in the ${}^3\mathrm{S}_1$ state, is defined as

$$M(n(\vec k,\alpha_1)\Sigma^0(-\vec k,\alpha_2)\to K^-(\vec 0)p(\vec K,\sigma_p)n(-\vec K,\sigma_n);{}^3\mathrm{P}_1) = iC^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \times\frac{[\bar u(\vec K,\sigma_p)\vec\gamma u^c(-\vec K,\sigma_n)]\cdot[\bar{u^c}(\vec k,\alpha_1)\vec\gamma\gamma^5 u(-\vec k,\alpha_2)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2-ia^t_{np}K} \times f^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0), \quad (7.5)$$

where $f^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)$ is the amplitude, describing the $n\Sigma^0$ rescattering in the ${}^3\mathrm{P}_1$ state near threshold of the $K^-(pn)_{^3\mathrm{S}_1}$ system production, and $C^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$ is the effective coupling constant of the transition $(n\Sigma^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$.

Using (7.2) and (7.3) we obtain the effective Lagrangian of the transition $(n\Lambda^0)_{^3\mathrm{P}_1}\to K^-(pn)_{^3\mathrm{S}_1}$ near threshold:

$$\mathcal{L}^{(n\Sigma^0;{}^3\mathrm{P}_1)\to K^-(pn;{}^3\mathrm{S}_1)}_{\mathrm{eff}}(x) = iC^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}K^{-\dagger}(x)[\bar p(x)\vec\gamma n^c(x)]\cdot[\bar{n^c}(x)\vec\gamma\gamma^5\Sigma^0(x)]. \quad (7.6)$$

The effective coupling constant $C^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$ is equal to[16]

$$C^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} = -7\times10^{-7}\,\mathrm{MeV}^{-3}, \quad (7.7)$$

where $E_{\Sigma^0}=1302\,\mathrm{MeV}$, $E_N=1072\,\mathrm{MeV}$ and $k_0=516\,\mathrm{MeV}$.

The amplitude $f^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)$, describing the rescattering of the $n\Sigma^0$ pair in the ${}^3\mathrm{P}_1$ state near threshold of the $K^-(pn)_{^3\mathrm{S}_1}$ system production, is defined by the Feynman diagrams depicted in fig. 6 with a replacement $\Lambda^0\to\Sigma^0$ and reads (see (6.11))

$$\left|f^{(n\Sigma^0;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\right| = \left|\left\{1-\frac{C_{n\Sigma^0}({}^3\mathrm{P}_1)}{12\pi^2}\frac{k_0^3}{E(k_0)}\left[\ln\left(\frac{E(k_0)+k_0}{E(k_0)-k_0}\right)-i\pi\right]\right\}^{-1}\right| \simeq 1, \quad (7.8)$$

where $E(k_0)=\sqrt{k_0^2+m_B^2}$[17] and the effective coupling constant $C_{n\Sigma^0}({}^3\mathrm{P}_1)$ is equal to

$$C_{n\Sigma^0}({}^3\mathrm{P}_1) = (2\alpha-1)^2\frac{g^2_{\pi NN}}{4k_0^2}\ln\left(1+\frac{4k_0^2}{m_K^2}\right) +\alpha(3-4\alpha)\frac{g^2_{\pi NN}}{6k_0^2}\ln\left(1+\frac{4k_0^2}{m_\eta^2}\right) = 0.7\times10^{-4}\,\mathrm{MeV}^{-2}. \quad (7.9)$$

[16] The analytical expression in given in [11].

[17] For simplicity we use the equal masses of baryons for the calculation of the rescattering of the $n\Sigma^0$ pair, where $m_B=\sqrt{(2m_N+m_K)^2-4k_0^2}/2=1070\,\mathrm{MeV}$.

The rescattering of the $n\Sigma^0$ pair in the ${}^3\mathrm{P}_1$ state is defined by the interaction, computed in the one-meson exchange approximation (see [19]):

$$\mathcal{L}_{\rm eff}^{(n\Sigma^0;{}^3\mathrm{P}_1)\to(n\Sigma^0;{}^3\mathrm{P}_1)}(x) = -\frac{1}{4}C_{n\Sigma^0}({}^3\mathrm{P}_1)[\bar{\Sigma}^0(x)\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Sigma^0(x)]. \quad (7.10)$$

In our approach the effective Lagrangian (7.10) describes also the final-state $(n\Sigma^0)_{{}^3\mathrm{P}_1}$ interaction near threshold of the reaction $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Sigma^0)_{{}^3\mathrm{P}_1}$.

7.3 S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)}$ of K^-d scattering

The amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)}$ can be computed similar to $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^3\mathrm{P}_1)}$. The summation over polarizations of the coupled baryons is defined by (6.15). This gives the imaginary part of the amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)}$ equal to

$$\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)} = 4.6\times10^{-3}\frac{1}{3\pi^2}\frac{m_\pi^3}{1+m_K/m_d}\frac{k_0^3}{2m_N+m_K} \times\Big[C_{K^-pn}^{(n\Sigma^0;{}^3\mathrm{P}_1)}\Big]^2\Big|f_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^3\mathrm{P}_1)}(k_0)\Big|^2 = 1.9\times10^{-3}\,\mathrm{fm}. \quad (7.11)$$

The real part of $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)}$ reads

$$\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)} = 4.6\times10^{-3}\frac{1}{12\pi^2}\frac{m_\pi^3}{1+m_K/m_d} \times\frac{k_0^2}{1+m_K/2m_B}\Big[C_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^3\mathrm{P}_1)}\Big]^2 \times\Big|f_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^3\mathrm{P}_1)}(k_0)\Big|^2 F\Big(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\Big) = 0.05\times10^{-3}\,\mathrm{fm}, \quad (7.12)$$

where $\Lambda = m_N$ and $m_B = \sqrt{(2m_N+m_K)^2-4k_0^2}/2 = 1070\,\mathrm{MeV}$.

The S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)}$ of K^-d scattering near threshold, caused by the two-body inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1} \to (n\Sigma^0)_{{}^3\mathrm{P}_1} \to K^-(pn)_{{}^3\mathrm{S}_1}$, is given by

$$\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)} = (0.05 + i1.90)\times10^{-3}\,\mathrm{fm}. \quad (7.13)$$

Now we proceed to computing the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^1\mathrm{P}_1)}$ of K^-d scattering near threshold, saturated by the intermediate $(n\Sigma^0)_{{}^1\mathrm{P}_1}$ state.

7.4 Reaction $(n\Sigma^0)_{{}^1\mathrm{P}_1} \to K^-(pn)_{{}^3\mathrm{S}_1}$

The amplitude of the reaction $(n\Sigma^0)_{{}^1\mathrm{P}_1} \to K^-(pn)_{{}^3\mathrm{S}_1}$ is defined by

$$M(n(\vec{k},\alpha_1)\Sigma^0(-\vec{k},\alpha_2)\to K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^1\mathrm{P}_1) = iC_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)} \times\frac{[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)]\cdot[\bar{u^c}(\vec{k},\alpha_1)\vec{\gamma}\gamma^5 u(-\vec{k},\alpha_2)]}{1-\frac{1}{2}r_{np}^t a_{np}^t K^2 + i a_{np}^t K} \times f_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)}(k_0). \quad (7.14)$$

The effective Lagrangian of the transition $(n\Sigma^0)_{{}^1\mathrm{P}_1} \to K^-(pn)_{{}^3\mathrm{S}_1}$ at threshold can be defined by

$$\mathcal{L}_{\rm eff}^{(n\Sigma^0;{}^1\mathrm{P}_1)\to K^-(pn;{}^3\mathrm{S}_1)}(x) = iC_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)}K^{-\dagger}(x)[\bar{p}(x)\vec{\gamma}n^c(x)] \cdot[\bar{n^c}(x)\gamma^0\vec{\gamma}\gamma^5\Sigma^0(x)]. \quad (7.15)$$

Using the effective Lagrangians (7.2) and (7.3), and the prescription for the projection of the four-baryon operators onto the ${}^3\mathrm{S}_1\otimes{}^3\mathrm{P}_1$ and ${}^3\mathrm{S}_1\otimes{}^1\mathrm{P}_1$ states (7.4) we obtain the effective coupling constant $C_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)}$ equal to

$$C_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Lambda^0;{}^1\mathrm{P}_1)} = -2\times10^{-7}\,\mathrm{MeV}^{-3}. \quad (7.16)$$

The Lagrangian (7.15) describes the interaction of the $n\Sigma^0$ pair in the ${}^1\mathrm{P}_1$ state with the np pair in the ${}^1\mathrm{S}_1$ through the emission of the K^--meson.

7.5 Amplitude of $(n\Sigma^0)_{{}^1\mathrm{P}_1}$ rescattering

The amplitude $f_{K^-pn}^{(n\Sigma^0;{}^1\mathrm{P}_1)}(k_0)$, describing the rescattering of the $n\Sigma^0$ pair in the ${}^1\mathrm{P}_1$ state near threshold of the K^-pn system production, is defined by the Feynman diagrams analogous to those depicted in fig. 6. The result of the calculation reads (see appendix E in [11])

$$\Big|f_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)}(k_0)\Big| = \left|\left\{1-\frac{C_{n\Sigma^0}({}^1\mathrm{P}_1)}{24\pi^2}\frac{k_0^3}{E(k_0)}\left[\ln\left(\frac{E(k_0)+k_0}{E(k_0)-k_0}\right)-i\pi\right]\right\}^{-1}\right| \simeq 1, \quad (7.17)$$

where $E(k_0) = \sqrt{k_0^2+m_B^2}$[18] and the effective coupling constant $C_{(n\Lambda^0)}({}^1\mathrm{P}_1)$ is equal to

$$C_{n\Sigma^0}({}^3\mathrm{P}_1) = C_{(n\Lambda^0)}({}^1\mathrm{P}_1) = 0.7\times10^{-4}\,\mathrm{MeV}^{-2}. \quad (7.18)$$

[18] For simplicity we use the equal masses of baryons for the calculation of the rescattering of the $n\Lambda^0$ pair, where $m_B = \sqrt{(2m_N+m_K)^2-4k_0^2}/2 = 1070\,\mathrm{MeV}$.

The rescattering of the $n\Sigma^0$ pair in the ${}^3\mathrm{P}_1$ state is defined by the interaction

$$\mathcal{L}_{\rm eff}^{(n\Sigma^0;{}^3\mathrm{P}_1)\to(n\Sigma^0;{}^3\mathrm{P}_1)}(x) = -\frac{1}{4}C_{n\Sigma^0}({}^3\mathrm{P}_1)[\bar{\Sigma}^0(x)\vec{\gamma}\gamma^5 n^c(x)]\cdot[\bar{n^c}(x)\vec{\gamma}\gamma^5\Sigma^0(x)]. \quad (7.19)$$

The amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^1\mathrm{P}_1)}$, caused by the intermediate $(n\Sigma^0)_{{}^1\mathrm{P}_1}$ state we define as follows.

7.6 S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^1P_1)}$ of K^-d scattering

The result of the summation over polarizations of interacting baryons is given by (6.30). Hence, at threshold the imaginary part of the S-wave amplitude of K^-d scattering, caused by the intermediate $(n\Sigma^0)_{{}^3\mathrm{P}_1}$ state, is equal to

$$\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^1\mathrm{P}_1)} = 4.6\times 10^{-3}\frac{1}{6\pi^2}\frac{m_\pi^3}{1+m_K/m_d}\frac{k_0^3}{2m_N+m_K} \times\Big[C_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)}\Big]^2\Big|f_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Sigma^0;{}^1\mathrm{P}_1)}(k_0)\Big|^2 = 0.1\times 10^{-3}\,\mathrm{fm}. \quad (7.20)$$

The real part of $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)}$ reads

$$\mathcal{R}e\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^1\mathrm{P}_1)} = 4.6\times 10^{-3}\frac{1}{24\pi^2}\frac{m_\pi^3}{1+m_K/m_d}\frac{k_0^2}{1+m_K/2m_B} \times\Big[C_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Lambda^0;{}^1\mathrm{P}_1)}\Big]^2\Big|f_{K^-(pn;{}^3\mathrm{S}_1)}^{(n\Lambda^0;{}^1\mathrm{P}_1)}(k_0)\Big|^2 F\Big(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\Big) = 2\times 10^{-6}\,\mathrm{fm}. \quad (7.21)$$

Thus, the S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^1\mathrm{P}_1)}$ of K^-d scattering, caused by the two-body inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to(n\Sigma^0)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$, is equal to

$$\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^1\mathrm{P}_1)} = (0.02+i1.00)\times 10^{-4}\,\mathrm{fm}. \quad (7.22)$$

Now we can estimate the contribution of the two-body inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to n\Sigma^0\to K^-(pn)_{{}^3\mathrm{S}_1}$ to the S-wave amplitude $\tilde{f}_0^{K^-d}(0)$ of K^-d scattering near threshold and the energy level displacement of the ground state of kaonic deuterium.

7.7 S-wave amplitude $f_0^{K^-d}(0)_{n\Sigma^0}$ and the energy level displacement

The S-wave amplitude of K^-d scattering at threshold, saturated by the inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to n\Sigma^0\to K^-(pn)_{{}^3\mathrm{S}_1}$ with the $n\Sigma^0$ pair in the ${}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ state, is equal to the sum of the contributions (7.13) and (7.22)

$$\tilde{f}_0^{K^-d}(0)_{n\Sigma^0} = (0.05+i2.00)\times 10^{-3}\,\mathrm{fm}. \quad (7.23)$$

The contribution of the decay $A_{Kd}\to n\Lambda^0$ to the energy level displacement of the ground state of kaonic deuterium amounts to

$$-\epsilon_{1s}^{(n\Sigma^0)} + i\frac{\Gamma_{1s}^{(n\Sigma^0)}}{2} = 602\tilde{f}_0^{K^-d}(0)_{n\Sigma^0} = (0.03+i1.2)\,\mathrm{eV}. \quad (7.24)$$

Hence, the partial width of the decay $A_{Kd}\to n\Lambda^0$ is equal to $\Gamma_{1s}^{(n\Sigma^0)} = 2.4\,\mathrm{eV}$.

According to [31], the experimental rate of the production of the $n\Sigma^0$ pair at threshold of the reaction $K^-d\to n\Sigma^0$ is equal to $R(K^-d\to n\Sigma^0) = (0.337\pm 0.070)\%$.

Using our estimate of the partial width, $\Gamma_{1s}^{(n\Sigma^0)} = (2.40\pm 0.48)\,\mathrm{eV}$, where $\pm 0.48\,\mathrm{eV}$ is a theoretical accuracy of the result, and the experimental rate, $R(K^-d\to n\Sigma^0) = (0.337\pm 0.070)\%$, we can estimate the expected value of the total width of the energy level of the ground state of kaonic deuterium

$$\Gamma_{1s} = \frac{\Gamma_{1s}^{(n\Sigma^0)}}{(0.337\pm 0.070)\times 10^{-2}} = (700\pm 200)\,\mathrm{eV}. \quad (7.25)$$

This value is compared well with our estimate $\Gamma_{1s} = (570\pm 130)\,\mathrm{eV}$, which we have made in sect. 6 using the theoretical value of the partial width of the decay $A_{Kd}\to n\Lambda^0$ and the experimental rate of the $n\Lambda^0$ production in the reaction $K^-d\to n\Lambda^0$.

8 Amplitude of reaction $K^-(pn)_{{}^3S_1}\to p\Sigma^-\to K^-(pn)_{{}^3S_1}$ and the energy level displacement

The amplitudes of the reactions $K^-(pn)_{{}^3\mathrm{S}_1}\to(p\Sigma^-)_X$, where the $p\Sigma^-$ pair couples in the $X={}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ state, we define as

$$M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to p(\vec{k},\alpha_1)\Sigma^-(-\vec{k},\alpha_2);{}^3\mathrm{P}_1) = -iC_{K^-(pn;{}^3\mathrm{S}_1)}^{(p\Sigma^-;{}^3\mathrm{P}_1)} \times\frac{\big[\bar{u^c}(-\vec{K},\sigma_n)\vec{\gamma}u(\vec{K},\sigma_p)\big]\cdot\big[\bar{u}(-\vec{k},\alpha_2)\vec{\gamma}\gamma^5u^c(\vec{k},\alpha_1)\big]}{1-\frac{1}{2}r_{np}^t a_{np}^t K^2 + ia_{np}^t K} \times f_{K^-(pn;{}^3\mathrm{S}_1)}^{(p\Sigma^-;{}^3\mathrm{P}_1)}(k_0),$$

$$M(K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n)\to p(\vec{k},\alpha_1)\Sigma^-(-\vec{k},\alpha_2);{}^1\mathrm{P}_1) = -iC_{K^-(pn;{}^3\mathrm{S}_1)}^{(p\Sigma^-;{}^1\mathrm{P}_1)} \times\frac{\big[\bar{u^c}(-\vec{K},\sigma_n)\vec{\gamma}u(\vec{K},\sigma_p)\big]\cdot\big[\bar{u}(-\vec{k},\alpha_2)\gamma^0\vec{\gamma}\gamma^5u^c(\vec{k},\alpha_1)\big]}{1-\frac{1}{2}r_{np}^t a_{np}^t K^2 + ia_{np}^t K} \times f_{K^-(pn;{}^3\mathrm{S}_1)}^{(p\Sigma^-;{}^1\mathrm{P}_1)}(k_0), \quad (8.1)$$

where $f_{K^-(pn;{}^3\mathrm{S}_1)}^{(p\Sigma^-;X)}(k_0)$ is the amplitude of the final-state $p\Sigma^-$ interaction near threshold of the reaction $K^-(pn)_{{}^3\mathrm{S}_1}\to(p\Sigma^-)_X$ and $C_{K^-(pn;{}^3\mathrm{S}_1)}^{(p\Sigma^0;X)}$ is the effective coupling constant of the transition $K^-(pn)_{{}^3\mathrm{S}_1}\to(p\Sigma^-)_X$, where $X={}^3\mathrm{P}_1$ or ${}^1\mathrm{P}_1$.

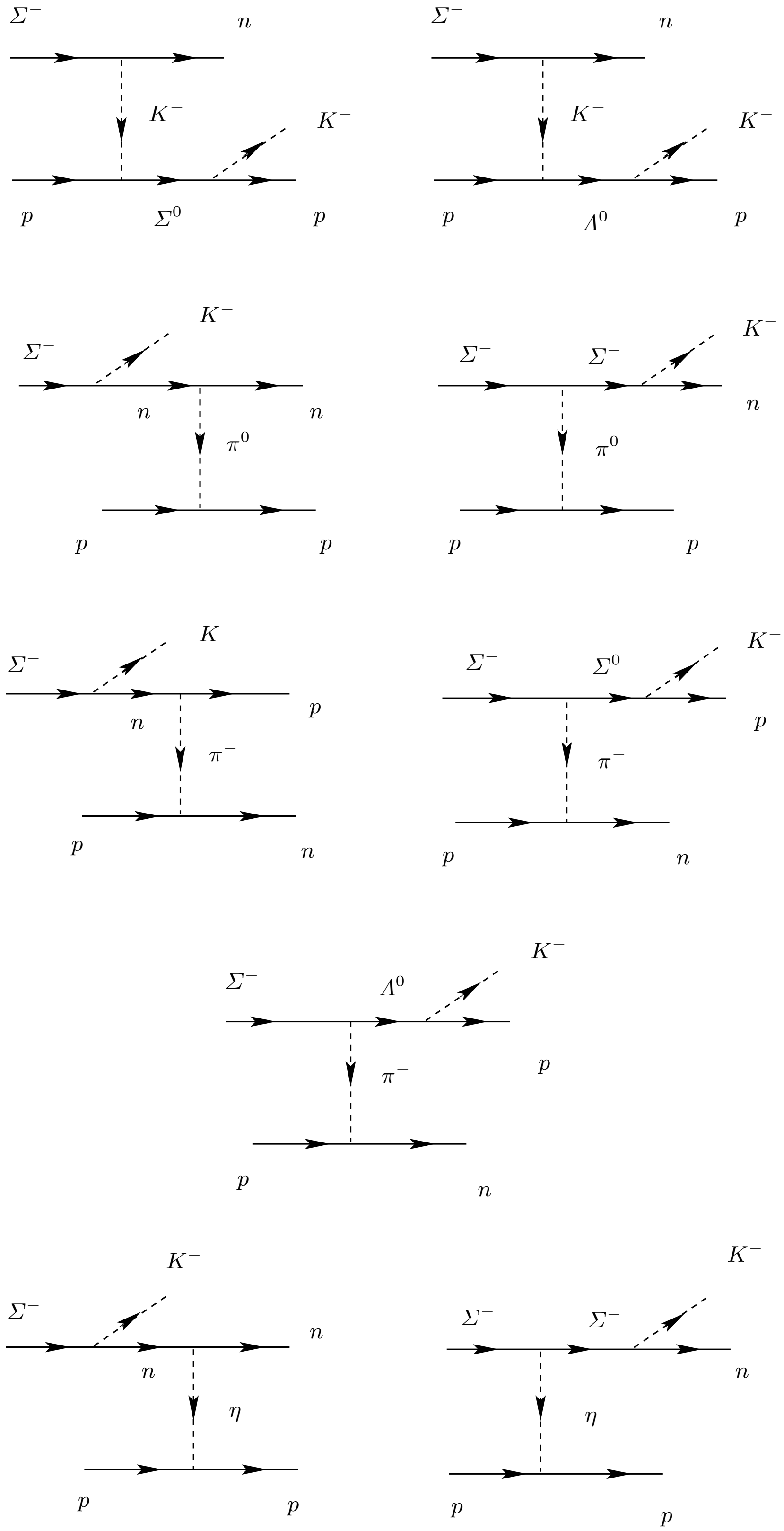

Fig. 9. Feynman diagrams describing the effective coupling constant $C^{p\Sigma^-}_{K^-pn}$ of the transition $p\Sigma^- \to K^-pn$ in the one-pseudoscalar-meson exchange approximation.

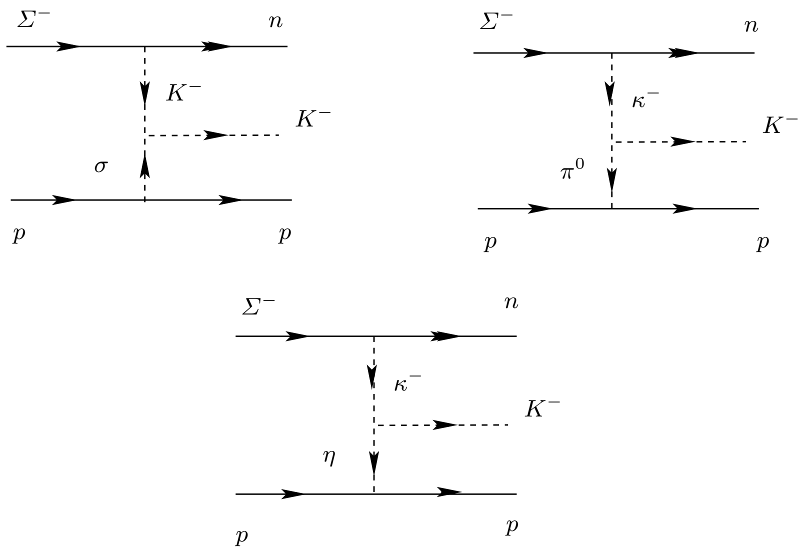

Fig. 10. Feynman diagrams describing the contribution of scalar mesons to the effective coupling constant of the transition $p\Sigma^- \to K^- pn$ in the one-pseudoscalar-meson exchange approximation.

8.1 Effective coupling constant $C^{p\Sigma^-}_{K^- pn}$

The transition $p\Sigma^- \to K^- pn$, induced by the one-pseudoscalar-meson exchange, is defined by the set of Feynman diagrams depicted in fig. 9. The Feynman diagrams for the transition $p\Sigma^- \to K^- pn$, determined in the one-pseudoscalar-meson approximation with scalar-meson exchanges, we adduce in fig. 10. In the momentum representation the amplitude of the transition $p\Sigma^- \to K^- pn$, determined by the Feynman diagrams in fig. 9, is given in [11]. Near threshold of the transition $p\Sigma^- \to K^- pn$ the set of diagrams in fig. 9 we represent in the form of the local effective Lagrangian

$$
\begin{aligned}
&\mathcal{L}^{p\Sigma^- \to K^- pn}_{\rm eff}(x)_P = \\
&-\sqrt{2}\frac{(2\alpha-1)^3 g^3_{\pi NN}}{m^2_K - (E_{\Sigma^-} - m_N)^2 + k^2_0} \\
&\times \frac{1}{m_\Sigma + m_N + m_K}[\bar{n}(x) i\gamma^5 \Sigma^-(x)][\bar{p}(x) p(x)] \\
&+\frac{\sqrt{2}}{3}\frac{(2\alpha-1)(3-2\alpha)^2 g^3_{\pi NN}}{m^2_K - (E_{\Sigma^-} - m_N)^2 + k^2_0} \\
&\times \frac{1}{m_{\Lambda^0} + m_N + m_K}[\bar{n}(x) i\gamma^5 \Sigma^-(x)][\bar{p}(x) p(x)] \\
&+\sqrt{2}\frac{(2\alpha-1) g^3_{\pi NN}}{m^2_\pi - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{m_N + m_K - m_\Sigma}{m^2_N - (E_{\Sigma^-} - m_K)^2 + k^2_0} \\
&\times [\bar{n}(x)\Sigma^-(x)][\bar{p}(x) i\gamma^5 p(x)] \\
&+2\sqrt{2}\frac{(1-\alpha)(2\alpha-1) g^3_{\pi NN}}{m^2_\pi - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{1}{m_\Sigma + m_N + m_K}[\bar{n}(x)\Sigma^-(x)][\bar{p}(x) i\gamma^5 p(x)] \\
&-2\sqrt{2}\frac{(2\alpha-1) g^3_{\pi NN}}{m^2_\pi - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{m_N + m_K - m_\Sigma}{m^2_N - (E_{\Sigma^-} - m_K)^2 + k^2_0} \\
&\times [\bar{p}(x)\Sigma^-(x)][\bar{n}(x) i\gamma^5 p(x)] \\
&-2\sqrt{2}\frac{(1-\alpha)(2\alpha-1) g^3_{\pi NN}}{m^2_\pi - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{1}{m_\Sigma + m_N + m_K}[\bar{p}(x)\Sigma^-(x)][\bar{n}(x) i\gamma^5 p(x)] \\
&+\frac{2\sqrt{2}}{3}\frac{\alpha(3-2\alpha) g^3_{\pi NN}}{m^2_\pi - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{1}{m_{\Lambda^0} + m_N + m_K}[\bar{p}(x)\Sigma^-(x)][\bar{n}(x) i\gamma^5 p(x)] \\
&-\frac{\sqrt{2}}{3}\frac{(2\alpha-1) g^3_{\pi NN}}{m^2_\eta - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{m_N + m_K - m_\Sigma}{m^2_N - (E_{\Sigma^-} - m_K)^2 + k^2_0} \\
&\times [\bar{n}(x)\Sigma^-(x)][\bar{p}(x) i\gamma^5 p(x)] \\
&-\frac{2\sqrt{2}}{3}\frac{\alpha(2\alpha-1)(3-4\alpha) g^3_{\pi NN}}{m^2_\eta - (E_N - m_N)^2 + k^2_0} \\
&\times \frac{1}{m_\Sigma + m_N + m_K}[\bar{n}(x)\Sigma^-(x)][\bar{p}(x) i\gamma^5 p(x)], \quad (8.2)
\end{aligned}
$$

where $E_{\Sigma^-} = 1302\,\text{MeV}$, $E_N = 1072\,\text{MeV}$ and $k_0 = 516\,\text{MeV}$.

The effective Lagrangian of the transition $p\Sigma^- \to K^- pn$, defined by the Feynman diagrams in fig. 10 with the scalar-meson exchanges, is equal to

$$\mathcal{L}^{p\Sigma^-\to K^-pn}_{\rm eff}(x)_S = -\frac{1}{\sqrt{2}}\frac{1}{g_AF_\pi}\frac{(2\alpha-1)g^2_{\pi NN}}{m^2_K-(E_{\Sigma^-}-m_N)^2+k^2_0}\times[\bar{n}(x)i\gamma^5\Sigma^-(x)][\bar{p}(x)p(x)] + \frac{1}{\sqrt{2}}\frac{1}{g_AF_\pi}\frac{(2\alpha-1)g^2_{\pi NN}}{m^2_\pi-(E_N-m_N)^2+k^2_0}\times[\bar{n}(x)\Sigma^-(x)][\bar{p}(x)i\gamma^5p(x)] - \frac{1}{3\sqrt{2}}\frac{1}{g_AF_\pi}\frac{(2\alpha-1)(3-4\alpha)g^2_{\pi NN}}{m^2_\eta-(E_N-m_N)^2+k^2_0}\times[\bar{n}(x)\Sigma^-(x)][\bar{p}(x)i\gamma^5p(x)]. \quad (8.3)$$

Recall that the contribution of the scalar mesons is computed in the infinite mass limit corresponding to the non-linear realization of chiral symmetry. The effective Lagrangians defining the transitions $(p\Sigma^-)_{^3{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$ and $(p\Sigma^-)_{^1{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$ can be derived from (8.2) and (8.3) by means of the Fierz transformation (see (6.3) and (6.4))

$$[\bar{n}(x)\gamma^5\Sigma^-(x)][\bar{p}(x)p(x)]\to +\frac{1}{4}[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\vec{\gamma}\gamma^5\Sigma^-(x)] - \frac{1}{4}[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\gamma^0\vec{\gamma}\gamma^5\Sigma^-(x)],$$
$$[\bar{n}(x)\Sigma^-(x)][\bar{p}(x)\gamma^5p(x)]\to -\frac{1}{4}[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\vec{\gamma}\gamma^5\Sigma^-(x)] - \frac{1}{4}[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\gamma^0\vec{\gamma}\gamma^5\Sigma^-(x)],$$
$$[\bar{p}(x)\Sigma^-(x)][\bar{n}(x)\gamma^5p(x)]\to +\frac{1}{4}[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\vec{\gamma}\gamma^5\Sigma^-(x)] + \frac{1}{4}[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\gamma^0\vec{\gamma}\gamma^5\Sigma^-(x)]. \quad (8.4)$$

Substituting (8.4) into (8.2) and (8.3), we obtain the effective Lagrangians responsible for the transitions $(p\Sigma^-)_{^3{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$ and $(p\Sigma^-)_{^1{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$.

8.2 Reaction $(\mathrm{p}\Sigma^-)_{^3\mathrm{P}_1}\to \mathrm{K}^-(\mathrm{pn})_{^3\mathrm{S}_1}$

The amplitude of the reaction $(p\Sigma^-)_{^3{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$ is defined by

$$M(p(\vec{k},\alpha_1)\Sigma^-(-\vec{k},\alpha_2)\to K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^3{\rm P}_1) = iC^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}\times\frac{[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)]\cdot[\bar{u^c}(\vec{k},\alpha_1)\vec{\gamma}\gamma^5u(-\vec{k},\alpha_2)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2-ia^t_{np}K}\times f^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}(k_0), \quad (8.5)$$

where $f^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}(k_0)$ is the amplitude, describing the $p\Sigma^-$ rescattering in the ${}^3{\rm P}_1$ state near threshold of the $K^-(pn)_{^3{\rm S}_1}$ system production and $C^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}$ is the effective coupling constant of the transition $(p\Sigma^-)_{^3{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$.

The effective Lagrangian of the transition $(p\Sigma^-)_{^3{\rm P}_1}\to K^-(pn)_{^3{\rm S}_1}$, computed at threshold, reads

$$\mathcal{L}^{(p\Sigma^-;{}^3{\rm P}_1)\to K^-(pn;{}^3{\rm S}_1)}_{\rm eff}(x)_P = iC^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}K^{-\dagger}(x)[\bar{n}(x)\vec{\gamma}p^c(x)]\cdot[\bar{p^c}(x)\vec{\gamma}\gamma^5\Sigma^-(x)]. \quad (8.6)$$

The effective coupling constant $C^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}$ is defined by [11]

$$C^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)} = -7\times10^{-7}\,{\rm MeV}^{-3}. \quad (8.7)$$

The amplitude $f^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm S}_1)}(k_0)$, describing the rescattering of the $p\Sigma^-$ pair in the ${}^3{\rm P}_1$ state near threshold of the $K^-(pn)_{^3{\rm S}_1}$ system production, is defined by the Feynman diagrams similar to those depicted in fig. 6. The procedure of the calculation of these diagrams is expounded in appendix E in [11]. The result of the calculation reads

$$\Big|f^{(p\Sigma^-;{}^3{\rm P}_1)}_{K^-(pn;{}^3{\rm P}_1)}(k_0)\Big| = \left|\left\{1-\frac{C_{p\Sigma^-}({}^3{\rm P}_1)}{12\pi^2}\frac{k^3_0}{E(k_0)}\left[\ln\left(\frac{E(k_0)+k_0}{E(k_0)-k_0}\right)-i\pi\right]\right\}^{-1}\right| \simeq 0.6, \quad (8.8)$$

where $E(k_0)=\sqrt{k^2_0+m^2_B}$[19] and the effective coupling constant $C_{p\Sigma^-}({}^3{\rm P}_1)$ is equal to

$$C_{p\Sigma^-}({}^3{\rm P}_1) = -(1-\alpha)\frac{g^2_{\pi NN}}{2k^2_0}\ln\left(1+\frac{4k^2_0}{m^2_\pi}\right) + \alpha(3-4\alpha)\frac{g^2_{\pi NN}}{6k^2_0}\ln\left(1+\frac{4k^2_0}{m^2_\eta}\right) = -4.0\times10^{-4}\,{\rm MeV}^{-2}. \quad (8.9)$$

The rescattering of the $p\Sigma^-$ pair in the ${}^3{\rm P}_1$ state is defined by the interaction, computed in the one-meson exchange approximation

$$\mathcal{L}^{(p\Sigma^-;{}^3{\rm P}_1)\to(p\Sigma^-;{}^3{\rm P}_1)}_{\rm eff}(x) = -\frac{1}{4}C_{p\Sigma^-}({}^3{\rm P}_1)[\bar{\Sigma}^-(x)\vec{\gamma}\gamma^5p^c(x)]\cdot[\bar{p^c}(x)\vec{\gamma}\gamma^5\Sigma^-(x)]. \quad (8.10)$$

We would like to remind that the interaction (8.10) defines also the final-state $(p\Sigma^-)_{^3{\rm P}_1}$ interaction near threshold of the reaction $K^-(pn)_{^3{\rm P}_1}\to(p\Sigma^-)_{^3{\rm P}_1}$.

[19] For simplicity we use the equal masses of baryons for the calculation of the rescattering of the $p\Sigma^-$ pair, where $m_B=\sqrt{(2m_N+m_K)^2-4k^2_0}/2=1070\,{\rm MeV}$.

8.3 S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(p\Sigma^-;{}^3P_1)}$ of K^-d scattering

The amplitude $\tilde{f}_0^{K^-d}(0)_{(p\Sigma^-;{}^3\mathrm{P}_1)}$ can be computed in a way similar to $\tilde{f}_0^{K^-d}(0)_{(n\Lambda^0;{}^3\mathrm{P}_1)}$ and $\tilde{f}_0^{K^-d}(0)_{(n\Sigma^0;{}^3\mathrm{P}_1)}$. The result reads

$$\tilde{f}_0^{K^-d}(0)_{(p\Sigma^-;{}^3\mathrm{P}_1)} = 4.6\times 10^{-3}\frac{1}{3\pi^2}\frac{m_\pi^3}{1+m_K/m_d} \times\Big[C^{(p\Sigma^-;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}\Big]^2\Big|f^{(p\Sigma^-;{}^3\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\Big|^2 \times\left[\frac{1}{4}\frac{k_0^2}{1+m_K/2m_B}F\left(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\right)+i\frac{k_0^3}{2m_N+m_K}\right] = (0.02+i0.7)\times 10^{-3}\,\mathrm{fm}. \quad (8.11)$$

Now we proceed to computing the contribution of the reaction $(n\Sigma^0)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$.

8.4 Reaction $(p\Sigma^-)_{{}^1P_1}\to K^-(pn)_{{}^3S_1}$

The amplitude of the reaction $(p\Sigma^-)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$ is defined by

$$M(p(\vec{k},\alpha_1)\Sigma^-(-\vec{k},\alpha_2)\to K^-(\vec{0})p(\vec{K},\sigma_p)n(-\vec{K},\sigma_n);{}^1\mathrm{P}_1) = iC^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} \times\frac{[\bar{u}(\vec{K},\sigma_p)\vec{\gamma}u^c(-\vec{K},\sigma_n)]\cdot[\bar{u^c}(\vec{k},\alpha_1)\vec{\gamma}\gamma^5u(-\vec{k},\alpha_2)]}{1-\frac{1}{2}r^t_{np}a^t_{np}K^2+ia^t_{np}K} \times f^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0). \quad (8.12)$$

The effective Lagrangian of the transition $(p\Sigma^-)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$ at threshold can be defined by

$$\mathcal{L}_{\mathrm{eff}}^{(p\Sigma^-;{}^1\mathrm{P}_1)\to K^-(pn;{}^3\mathrm{S}_1)}(x) = iC^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}K^{-\dagger}(x)[\bar{n}(x)\vec{\gamma}p^c(x)] \cdot[\bar{p^c}(x)\gamma^0\vec{\gamma}\gamma^5\Sigma^-(x)]. \quad (8.13)$$

Using (8.2), (8.3) and (8.4) we compute the effective coupling constant $C^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}$. It is equal to

$$C^{(n\Lambda^0;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)} = -12\times 10^{-7}\,\mathrm{MeV}^{-3}. \quad (8.14)$$

The Lagrangian (8.13) describes the interaction of the $p\Sigma^-$ pair in the ${}^1\mathrm{P}_1$ state with the np pair in the ${}^1\mathrm{S}_1$ state through the emission of the K^--meson.

8.5 Amplitude of $(p\Sigma^-)_{{}^1P_1}$ rescattering

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The amplitude $f^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)$, describing the rescattering of the $n\Sigma^0$ pair in the ${}^1\mathrm{P}_1$ state near threshold of the K^-pn system production, is given by (see appendix E in [11])

$$\Big|f^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\Big| = \left|\left\{1-\frac{C_{p\Sigma^-}({}^1\mathrm{P}_1)}{24\pi^2}\frac{k_0^3}{E(k_0)}\left[\ln\left(\frac{E(k_0)+k_0}{E(k_0)-k_0}\right)-i\pi\right]\right\}^{-1}\right| \simeq 0.8. \quad (8.15)$$

The effective coupling constant $C_{p\Sigma^-}({}^1\mathrm{P}_1)$ is equal to

$$C_{p\Sigma^-}({}^1\mathrm{P}_1) = C_{p\Sigma^-}({}^3\mathrm{P}_1) = -4.0\times 10^{-4}\,\mathrm{MeV}^{-2}. \quad (8.16)$$

The rescattering of the $p\Sigma^-$ pair in the ${}^3\mathrm{P}_1$ state is defined by the interaction

$$\mathcal{L}_{\mathrm{eff}}^{(p\Sigma^-;{}^1\mathrm{P}_1)\to(p\Sigma^-;{}^1\mathrm{P}_1)}(x) = -\frac{1}{4}C_{p\Sigma^-}({}^1\mathrm{P}_1)[\bar{\Sigma}^-(x)\vec{\gamma}\gamma^5p^c(x)] \cdot[\bar{p^c}(x)\vec{\gamma}\gamma^5\Sigma^-(x)]. \quad (8.17)$$

Recall that according to [19] the effective coupling constant $C_{p\Sigma^-}({}^1\mathrm{P}_1)$ is computed in the one-pseudoscalar-meson exchange approximation.

8.6 S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(p\Sigma^-;{}^1P_1)}$ of K^-d scattering

The S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(p\Sigma^-;{}^1\mathrm{P}_1)}$ of K^-d scattering near threshold, caused by the reaction $K^-(pn)_{{}^3\mathrm{S}_1}\to(p\Sigma^-)_{{}^1\mathrm{P}_1}\to K^-(pn)_{{}^3\mathrm{S}_1}$, is equal to

$$\tilde{f}_0^{K^-d}(0)_{(p\Sigma^-;{}^1\mathrm{P}_1)} = 4.6\times 10^{-3}\frac{1}{6\pi^2} \times\frac{m_\pi^3}{1+m_K/m_d}\Big[C^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}\Big]^2\Big|f^{(p\Sigma^-;{}^1\mathrm{P}_1)}_{K^-(pn;{}^3\mathrm{S}_1)}(k_0)\Big|^2 \times\left[\frac{1}{4}\frac{k_0^2}{1+m_K/2m_B}F\left(\frac{\Lambda}{m_B},\frac{k_0}{m_B}\right)+i\frac{k_0^3}{2m_N+m_K}\right] = (0.05+i1.8)\times 10^{-3}\,\mathrm{fm}. \quad (8.18)$$

Now we can compute the contribution of the two-body inelastic channel $K^-(pn)_{{}^3\mathrm{S}_1}\to p\Sigma^-\to K^-(pn)_{{}^3\mathrm{S}_1}$ to the S-wave amplitude $f_0^{K^-d}(0)_{p\Sigma^-}$ of K^-d scattering near threshold and the energy level displacement of the ground state of kaonic deuterium.

8.7 S-wave amplitude $f_0^{K^-d}(0)_{p\Sigma^-}$ and the energy level displacement

The S-wave amplitude of K^-d scattering at threshold, saturated by the inelastic reaction $K^-(pn)_{{}^3\mathrm{S}_1}\to p\Sigma^-\to K^-(pn)_{{}^3\mathrm{S}_1}$ with the $p\Sigma^-$ pair in the ${}^3\mathrm{P}_1$ and ${}^1\mathrm{P}_1$ state, is equal to the sum of the contributions (8.11) and (8.18):

$$\tilde{f}_0^{K^-d}(0)_{p\Sigma^-} = (0.07+i2.5)\times 10^{-3}\,\mathrm{fm}. \quad (8.19)$$

The contribution of the decay $A_{Kd} \to p\Sigma^-$ to the energy level displacement of the ground state of kaonic deuterium amounts to

$$-\epsilon_{1s}^{(p\Sigma^-)} + i\frac{\Gamma_{1s}^{(n\Sigma^0)}}{2} = 602\tilde{f}_0^{K^-d}(0)_{n\Sigma^0} = (0.04 + i1.5)\,\mathrm{eV}. \quad (8.20)$$

Hence, the partial width of the decay $A_{Kd} \to p\Lambda^-$ is equal to $\Gamma_{1s}^{(p\Sigma^-)} = 3.0\,\mathrm{eV}$.

According to [31], the experimental rate of the production of the $n\Sigma^0$ pair at threshold of the reaction $K^-d \to p\Sigma^-$ is equal to $R(K^-d \to p\Sigma^-) = (0.505 \pm 0.036)\%$.

Using our estimate of the partial width, $\Gamma_{1s}^{(p\Sigma^-)} = (3.0 \pm 0.6)\,\mathrm{eV}$, where $\pm 0.6\,\mathrm{eV}$ is a theoretical accuracy of the result, and the experimental rate, $R(K^-d \to n\Sigma^0) = (0.505 \pm 0.036)\%$, we can estimate the expected value of the total width of the energy level of the ground state of kaonic deuterium

$$\Gamma_{1s} = \frac{\Gamma_{1s}^{(p\Sigma^-)}}{(0.505 \pm 0.036) \times 10^{-2}} = (590 \pm 130)\,\mathrm{eV}. \quad (8.21)$$

This value is compared well with our estimate $\Gamma_{1s} = (570 \pm 130)\,\mathrm{eV}$ and $\Gamma_{1s} = (700 \pm 200)\,\mathrm{eV}$, which we have made in sects. 6 and 7 using the theoretical values of the partial widths of the decays $A_{Kd} \to n\Lambda^0$ and $A_{Kd} \to n\Sigma^0$ and the experimental rates of the $n\Lambda^0$ and $n\sigma^0$ production in the reactions $K^-d \to n\Lambda^0$ and $K^-d \to n\Sigma^0$.

9 Comparison with experimental data and the energy level displacement

The imaginary parts of the amplitudes $\tilde{f}_0^{K^-d}(0)_{NY}$ with $NY = n\Lambda^0, n\Sigma^0$ and $p\Sigma^-$ are proportional to the cross-sections for the reactions $K^-d \to n\Lambda^0$, $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$ near threshold. According to the experimental data by Veirs and Burnstein [31], the production rates of NY pairs in the reactions $K^-d \to n\Lambda^0$, $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$ are equal to

$$\begin{aligned} R(K^-d \to n\Lambda^0) &= (0.387 \pm 0.041)\%, \\ R(K^-d \to n\Sigma^0) &= (0.337 \pm 0.070)\%, \\ R(K^-d \to p\Sigma^-) &= (0.505 \pm 0.036)\%, \\ R = \sum_{NY} R(K^-d \to NY) &= (1.229 \pm 0.090)\%. \end{aligned} \quad (9.1)$$

The ratios, independent on a total yield, read

$$\begin{aligned} R(\Lambda^0/\Sigma^0) &= \frac{R(K^-d \to n\Lambda^0)}{R(K^-d \to n\Sigma^0)} = (1.15 \pm 0.27), \\ R(\Sigma^0/\Sigma^-) &= \frac{R(K^-d \to n\Sigma^0)}{R(K^-d \to p\Sigma^-)} = (0.68 \pm 0.15), \\ R(\Lambda^0/\Sigma^-) &= \frac{R(K^-d \to n\Lambda^0)}{R(K^-d \to p\Sigma^-)} = (0.77 \pm 0.10). \end{aligned} \quad (9.2)$$

For these ratios we predict the following theoretical values:

$$\begin{aligned} R(\Lambda^0/\Sigma^0) &= \frac{\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{n\Lambda^0}}{\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{n\Sigma^0}} = 1.0 \pm 0.3, \\ R(\Sigma^0/\Sigma^-) &= \frac{\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{n\Sigma^0}}{\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{p\Sigma^-}} = 0.8 \pm 0.2, \\ R(\Lambda^0/\Sigma^-) &= \frac{\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{n\Lambda^0}}{\mathcal{I}m\tilde{f}_0^{K^-d}(0)_{p\Sigma^-}} = 0.8 \pm 0.2. \end{aligned} \quad (9.3)$$

The theoretical predictions agree well with the experimental data.

We would like to emphasize that according to the requirement of isospin invariance the ratio $R(\Sigma^0/\Sigma^-)$ of the cross-sections for the reactions $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$ should be equal to

$$R(\Sigma^0/\Sigma^-) = 0.5. \quad (9.4)$$

We would like to notice that the strength of the forces responsible for the transitions $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$ is of order of a strength of the forces violating isospin invariance. Indeed, relative mass differences of the neutron and proton $(m_n - m_p)/m_N = 0.138\%$ and the charged and neutral K-mesons $(m_{\bar{K}^0} - m_{K^-})/m_K = 0.607\%$ are of order of the production rates of the NY pairs near threshold of the reactions $K^-d \to NY$. Therefore, a departure from the isospin invariance for the ratio of the cross-sections of the reactions $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$ should not be a surprise [31].

The contribution of the inelastic two-body channels $K^-d \to NY$ to the energy level displacement of the ground state of kaonic deuterium is given by

$$\begin{aligned} -\epsilon_{1s}^{(n\Lambda^0)} + i\frac{\Gamma_{1s}^{(n\Lambda^0)}}{2} &= 602\tilde{f}_0^{K^-d}(0)_{n\Lambda^0} = \\ &\quad (-0.10 \pm 0.02) + i(1.1 \pm 0.2)\,\mathrm{eV}, \\ -\epsilon_{1s}^{(n\Sigma^0)} + i\frac{\Gamma_{1s}^{(n\Sigma^0)}}{2} &= 602\tilde{f}_0^{K^-d}(0)_{n\Sigma^0} = \\ &\quad (+0.03 \pm 0.01) + i(1.2 \pm 0.3)\,\mathrm{eV}, \\ -\epsilon_{1s}^{(p\Sigma^-)} + i\frac{\Gamma_{1s}^{(p\Sigma^-)}}{2} &= 602\tilde{f}_0^{K^-d}(0)_{p\Sigma^-} = \\ &\quad (+0.04 \pm 0.01) + i(1.5 \pm 0.3)\,\mathrm{eV}. \end{aligned} \quad (9.5)$$

The partial widths $\Gamma_{1s}^{(NY)}$, equal to

$$\begin{aligned} \Gamma_{1s}^{(n\Lambda^0)} &= (2.2 \pm 0.5)\,\mathrm{eV}, \\ \Gamma_{1s}^{(n\Sigma^0)} &= (2.4 \pm 0.5)\,\mathrm{eV}, \\ \Gamma_{1s}^{(p\Sigma^-)} &= (3.0 \pm 0.6)\,\mathrm{eV}, \end{aligned} \quad (9.6)$$

compare well with the theoretical estimates discussed by Reitan [30]:

$$\begin{aligned} 0.66\,\mathrm{eV} &\le \begin{pmatrix} \Gamma_{1s}^{(n\Lambda^0)} \\ \Gamma_{1s}^{(n\Sigma^0)} \end{pmatrix} \le 190\,\mathrm{eV}, \\ 0.66\,\mathrm{eV} &\le \Gamma_{1s}^{(p\Sigma^-)} \le 3.95\,\mathrm{eV}. \end{aligned} \quad (9.7)$$

The S-wave amplitude $\tilde{f}_0^{K^-d}(0)_{(\text{two-body})}$ of K^-d scattering near threshold, caused by the two-body inelastic channels $K^-d \to NY \to K^-d$ with the intermediate $NY = n\Lambda^0, n\Sigma^0$ and $p\Sigma^-$ states, is equal to

$$\tilde{f}_0^{K^-d}(0)_{(\text{two-body})} = (-0.08 \pm 0.02) + i(6.4 \pm 0.8) \times 10^{-3}\,\text{fm}. \quad (9.8)$$

The energy level displacement of the ground state of kaonic deuterium caused by the inelastic two-body decays $A_{Kd} \to n\Lambda^0$, $A_{Kd} \to n\Sigma^0$ and $A_{Kd} \to p\Sigma^-$ is equal to

$$-\epsilon_{1s}^{(\text{two-body})} + i\frac{\Gamma_{1s}^{(\text{two-body})}}{2} = 602\tilde{f}_0^{K^-d}(0)_{(\text{two-body})} = (-0.05 \pm 0.01) + i(3.9 \pm 0.5)\,\text{eV}. \quad (9.9)$$

Using the experimental value of the total production rate $R = (1.229 \pm 0.090)\%$ and our theoretical prediction for $\Gamma_{1s}^{(\text{two-body})}$, given by (9.9), we can estimate the expected value of the total width of the ground state of kaonic deuterium

$$\Gamma_{1s} = \frac{\sum_{NY}\Gamma_{1s}^{(NY)}}{(1.229 \pm 0.090) \times 10^{-2}} = \frac{\Gamma_{1s}^{(n\Lambda^0)} + \Gamma_{1s}^{(n\Sigma^0)} + \Gamma_{1s}^{(p\Sigma^-)}}{(1.229 \pm 0.090) \times 10^{-2}} = \frac{7.8 \pm 1.0}{(1.229 \pm 0.090) \times 10^{-2}} = (630 \pm 100)\,\text{eV}. \quad (9.10)$$

This value agrees well with the estimates (6.36), (7.25) and (8.21).

Following the estimate of the total width, based on the theoretical values of the widths of the decays $A_{Kd} \to n\Lambda^0, n\Sigma^0$ and $A_{Kd} \to p\Sigma^-$ and the experimental rates of the reactions $K^-d \to n\Lambda^0$, $K^-d \to n\Sigma^0$ and $K^-d \to p\Sigma^-$, we can estimate the expected contribution of the three-body decays $A_{Kd} \to NY\pi$ to the shift of the energy level of the ground state of kaonic deuterium. We get

$$\epsilon_{1s}^{(\text{three-body})} = (9 \pm 8)\,\text{eV}. \quad (9.11)$$

This implies that the contribution of the inelastic channels with two-body $K^-d \to NY \to K^-d$ and three-body $K^-d \to NY\pi \to K^-d$ intermediate states to the real part of the S-wave amplitude of K^-d scattering near threshold is negligible small, and the real part of the S-wave amplitude of K^-d scattering near threshold is fully defined by the Ericson-Weise formula for the S-wave scattering length (4.17). This gives the following value for the shift of the energy level of the ground state of kaonic deuterium:

$$\epsilon_{1s} = -602\big(a_0^{K^-d}\big)_{\text{EW}} + \epsilon_{1s}^{(\text{three-body})} = (325 \pm 60)\,\text{eV}. \quad (9.12)$$

Thus, we predict that the S-wave amplitude of K^-d scattering near threshold is equal to

$$f_0^{K^-d}(0) = (-0.540 \pm 0.095) + i(0.521 \pm 0.075)\,\text{fm}. \quad (9.13)$$

This defines the energy level displacement of the ground state of kaonic deuterium

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = 602 f_0^{K^-d}(0) = (-325 \pm 60) + i(315 \pm 50)\,\text{eV}. \quad (9.14)$$

A confirmation of these estimates should go through the calculation of the contributions of the reactions $K^-(pn)_{{}^3\text{S}_1} \to NY\pi$ to the amplitude of low-energy elastic K^-d scattering.

10 Conclusion

The quantum field-theoretic and relativistic covariant approach, developed in [4–6] for the description of the energy level displacement of the ground and excited states of pionic hydrogen [4,5] and the energy level displacement of the ground state of kaonic hydrogen [6], has been applied to the analysis of the energy level displacement of the ground state of kaonic deuterium and the S-wave amplitude of elastic K^-d scattering near threshold.

According to [4–6] we have represented the energy level displacement of the ground state of kaonic deuterium in terms of the momentum integrals of the amplitude of elastic K^-d scattering for arbitrary relative momenta of the K^-d pair weighted with the wave functions of kaonic deuterium in the ground state. The knowledge of this amplitude should allow to compute the energy level displacement of the ground state of kaonic deuterium without any low-energy approximation. As has been shown in [4–6] the low-energy reduction of our representation of the energy level displacements of exotic atoms reproduces the well-known DGBT formula with additional corrections caused by the smearing of wave functions around the origin. Such a smearing is defined by the relativistic factors, related to the recoil energies of the nuclei [4]. These corrections are negative and of order 1%. They are universal for all exotic atoms, the existence of which is caused by Coulombic forces. Since the experimental accuracy of the measurement of the energy level displacement of the ground state of pionic hydrogen, reached recently by the PSI Collaboration [37], is of order 1%, the corrections, obtained in [4], play an important role for the correct extraction of the S-wave scattering lengths of πN scattering from the experimental data on the energy level displacement of the ground state of pionic hydrogen [38].

Since the experiments on the energy level displacement of the ground state of kaonic deuterium are in the stage of preparation, for the analyses of the energy level displacement of the ground state of kaonic deuterium, we can neglect the correction of order of 1% and use the DGBT formula.

In the analysis of the energy level displacement of the ground state of kaonic deuterium using the DGBT formula the main object of the theoretical investigation is the S-wave amplitude $f_0^{K^-d}(0)$ of K^-d scattering near threshold. As has been pointed out by Ericson and Weise [3] for the analysis of elastic low-energy π^-d scattering, the

S-wave scattering length of $\pi^- d$ scattering can be represented in the form of the superposition of the S-wave scattering lengths of $\pi^- p$ and $\pi^- n$ scattering, realizing so-called the *impulse approximation*, and the term, describing elastic $\pi^- pn$ scattering.

Following Ericson and Weise [3] and assuming that at threshold the S-wave amplitude of $K^- d$ scattering is defined by the superposition of the S-wave amplitudes of $K^- p$ and $K^- n$ scattering, reproducing the *impulse approximation*, and the S-wave amplitude of the three-body to three-body reaction $K^-(pn)_{^3S_1} \to K^-(pn)_{^3S_1}$, where the np pair couples in the $^3S_1$ state with isospin zero, we have introduced the wave function of the ground state of kaonic deuterium and the wave function of the deuteron in the momentum and the particle number representation in terms of the operators of creation of the K^--meson, the proton and the neutron. In appendix A in [11] we have shown that these wave functions describe the bound $K^- d$ state and the bound np state with quantum numbers of the deuteron.

We have shown that due to such a representation of the wave function of the deuteron, the S-wave amplitude of $K^- d$ scattering at threshold can be represented in the Ericson-Weise form. The real part of the S-wave amplitude of $K^- d$ scattering contains two terms, defined by the S-wave amplitudes of $K^- p$ and $K^- n$ scattering near threshold, and the terms coming from the interaction of three-body scattering $K^-(pn)_{^3S_1} \to K^-(pn)_{^3S_1}$. The imaginary part of the S-wave amplitude of $K^- d$ scattering near threshold is fully defined by S-wave amplitude of three-body scattering $K^-(pn)_{^3S_1} \to K^-(pn)_{^3S_1}$. The amplitudes of elastic $K^- p$, $K^- n$ and $K^-(pn)_{^3S_1}$ scattering are weighted with the wave functions of the deuteron in the momentum representation.

We would like to accentuate that the Ericson-Weise form of the S-wave scattering length of $\pi^- d$ scattering has been derived within a potential model approach. We have proved this form within the quantum field-theoretic approach [28] and have derived the $K^- d$ version of this formula. The main object of the Ericson-Weise formula, $\langle 1/r_{12}\rangle$, where $1/r_{12}$ is the inverse distance between the proton and the neutron averaged over all positions of them [3], we have computed equal to $\langle 1/r_{12}\rangle = 0.69\, m_\pi$ for $\pi^- d$ scattering [28] agreeing well with the Ericson-Weise value $\langle 1/r_{12}\rangle = 0.64\, m_\pi$ [3].

We have shown that in the case of $K^- d$ scattering the Ericson-Weise term, caused by elastic three-body $K^-(pn)_{^3S_1}$ scattering, is defined by the S-wave scattering lengths of $\bar{K}N$ scattering a^0_0 and a^1_0 with isospin zero and one, respectively, a^I_0 for $I = 0$ and $I = 1$. In our approach the real part $\mathcal{R}ef^{K^- d}_0(0)$ of the S-wave amplitude of $K^- d$ scattering near threshold is defined by the Ericson-Weise kind expression and the contribution of the inelastic channels of three-body reaction $K^-(pn)_{^3S_1} \to K^-(pn)_{^3S_1}$.

The term of the Ericson-Weise formula, defining the *impulse approximation*, takes the form of the superposition of the S-wave scattering length $a^{K^- p}_0$ of $K^- p$ scattering and the S-wave scattering length $a^{K^- n}_0$ of $K^- n$ scattering. The S-wave scattering length $a^{K^- p}_0$ of $K^- p$ scattering has been computed in [6].

For the calculation of the S-wave scattering length $a^{K^- n}_0$ of elastic $K^- n$ scattering we have followed [6]. We have represented the S-wave amplitude of $K^- n$ scattering near threshold in the form of the contribution of the $\Sigma^-(1750)$ resonance and the smooth elastic background. In our approach [6] the smooth elastic background is given by the contribution of defined by the low-energy interactions, which can be described by the Effective Chiral Lagrangians (ECL), and the exotic states such as the scalar mesons $a_0(980)$ and $f_0(980)$, which are the four-quarks states (or $K\bar{K}$ molecule), the description of which goes beyond the scope of ECL. Unlike $K^- p$ scattering, where the contribution of the exotic four-quark states $a_0(980)$ and $f_0(980)$ is very important for the correct description of the smooth elastic background, the scalar mesons $a_0(980)$ and $f_0(980)$ do not contribute to the real part of the S-wave amplitude of $K^- n$ scattering near threshold. As a result the smooth elastic background is fully determined by the contribution, described by ECL. We have computed the smooth elastic background for $K^- n$ scattering near threshold within the soft-kaon technique[20] and within the Effective chiral quark model with chiral $U(3)\times U(3)$ symmetry [18]. We have shown that these two approaches lead to the values of the smooth elastic background of $K^- n$ scattering which are compared within the accuracy about 10%.

This has made the result of the calculation of the smooth elastic background for $K^- p$ scattering, carried out in [6] within the Effective quark model with chiral $U(3) \times U(3)$ symmetry, more credible. Recall, that the calculation of the smooth elastic background for $K^- p$ scattering within the effective quark model with chiral $U(3) \times U(3)$ symmetry has been justified by the absence of the theoretical and experimental information about the coupling constants of the exotic scalar mesons $a_0(980)$ and $f_0(980)$ with nucleons. In [6] we have computed the smooth elastic background for $K^- p$ scattering near threshold within the Effective Chiral Lagrangian approach and fixed the coupling constants of the SNN interactions, where $S = a_0(980)$ and $f_0(980)$.

The imaginary part $\mathcal{I}mf^{K^- d}_0(0)$ of the S-wave amplitude of the three-body reaction $K^-(pn)_{^3S_1} \to K^-(pn)_{^3S_1}$ is determined by the inelastic channels with the two-body intermediate states $K^-(pn)_{^3S_1} \to NY \to K^-(pn)_{^3S_1}$, where NY is $n\Lambda^0$, $n\Sigma^0$ and $p\Sigma^-$, and the three-body intermediate states $K^-(pn)_{^3S_1} \to NY\pi \to K^-(pn)_{^3S_1}$.

We have computed the contributions of the two-body channels $K^-(pn)_{^3S_1} \to NY \to K^-(pn)_{^3S_1}$, where NY is $n\Lambda^0$, $n\Sigma^0$ and $p\Sigma^-$. The calculation of the amplitudes of the reactions $K^-(pn)_{^3S_1} \to NY$ we have carried out in the one-pseudoscalar and one-scalar meson exchange approximation. The contribution of scalar meson is computed in

[20] This is equivalent to the leading order in chiral expansion of ChPT by Gasser and Leutwyler with a non-linear realization of chiral $U(3)\times U(3)$ symmetry [35] realizing the ECL approach to the description of strong low-energy interactions of hadrons.

the infinite mass limit that corresponds to a non-linear realization of chiral $U(3) \times U(3)$ symmetry.

We have shown that the NY pairs in the reactions $K^-(pn)_{^3\mathrm{S}_1} \to NY$ can be produced in the $^3\mathrm{P}_1$ and $^1\mathrm{P}_1$ states. Accounting for the rescattering of the np pair in the $^3\mathrm{S}_1$ state and the NY pairs in the $^3\mathrm{P}_1$ and $^1\mathrm{P}_1$ states we have computed the S-wave amplitudes $f_0^{K^-d}(0)_{NY}$ of K^-d scattering near threshold, caused by the two-body inelastic channels $K^-(pn)_{^3\mathrm{S}_1} \to NY \to K^-(pn)_{^3\mathrm{S}_1}$. As has been pointed out in [19] the amplitudes of the rescattering of the NY pairs produced near threshold of the reactions $K^-(pn)_{^3\mathrm{S}_1} \to NY$ describe effectively the contribution of the set of resonances with the quantum numbers of the NY pairs.

Using the DGBT formula for the energy level displacement we have computed the energy level displacements of the ground state of kaonic deuterium induced by the two-body inelastic channels $K^-(pn)_{^3\mathrm{S}_1} \to NY \to K^-(pn)_{^3\mathrm{S}_1}$. Using the experimental data on the rates of the production of the states NY is $n\Lambda^0$, $n\Sigma^0$ and $p\Sigma^-$ in the reactions $K^-d \to NY$, we have estimated the expected value of the total width of the energy level of the ground state of kaonic deuterium and the contribution of the three-body inelastic channels to the shift of the energy level of the ground state of kaonic deuterium. As a result, we have found that the total energy level displacement of the ground state of kaonic deuterium should be equal to

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = 602 f_0^{K^-d}(0) = (-325 \pm 60) + i(315 \pm 50)\,\mathrm{eV},$$

that corresponds to the S-wave amplitude of K^-d scattering near threshold

$$f_0^{K^-d}(0) = (-0.540 \pm 0.095) + i(0.521 \pm 0.075)\,\mathrm{fm}.$$

The theoretical value of the S-wave amplitude of K^-d scattering near threshold and the approach, used for the calculation of the contribution of the inelastic two-body channels, have been justified by the description of the S-wave amplitude of π^-d scattering and the calculation of the contribution of the inelastic two-body channel $\pi^-d \to (nn)_{^3\mathrm{P}_1} \to \pi^-d$ in agreement with the experimental data on the energy level displacement of the ground state of pionic deuterium [28].

Of course, the complete confirmation of the theoretical prediction for the S-wave amplitude of K^-d scattering and the energy level displacement of the ground state of kaonic deuterium obtained above should go through the calculation of the contribution of the three-body inelastic channels, which we are planning to carry out in our forthcoming publication. Indeed, there are seven three-body inelastic channels $K^-(pn)_{^3\mathrm{S}_1} \to (NY)_{^3\mathrm{S}_1}\pi \to K^-(pn)_{^3\mathrm{S}_1}$ with $NY\pi = n\Sigma^-\pi^+, p\Sigma^-\pi^0, n\Sigma^0\pi^0, p\Sigma^0\pi^-, n\Sigma^+\pi^-, n\Lambda^0\pi^0$ and $p\Lambda^0\pi^-$, which would exceed a reasonable size of this paper.

The theoretical value of the energy level displacement of the ground state of kaonic deuterium obtained above can be used for the planning of experiments on the measurement of the energy level displacement of kaonic deuterium by the DEAR Collaboration at Frascati.

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